An alternative approach for satisfying stress constraints in continuum topology optimization using nonlinear material modeling

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The challenges of stress constraints

Characteristics of stress-constrained continuum topology optimization:

- Basic engineering requirement: remain linear-elastic
- Local measure $\rightarrow$ large number of constraints
- Removal of material $\rightarrow$ vanishing of constraint

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Large number of design variables, large number of constraints

Challenge #2: SINGULARITY

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Dealing with the challenge of complexity (1/2)

Strategy 1: consider all local constraints, solve with "active" subsets

[Duysinx and Bendsøe, 1998], [Bruggi and Duysinx, 2012]
[Pereira et al., 2004], [Fancello, 2006] - Augmented Lagrangian

Strategy 2: aggregate local constraints into global stress function, using K-S or p-norm functions

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Other approaches

[Amstutz and Novotny, 2010] - topological derivative, external penalty
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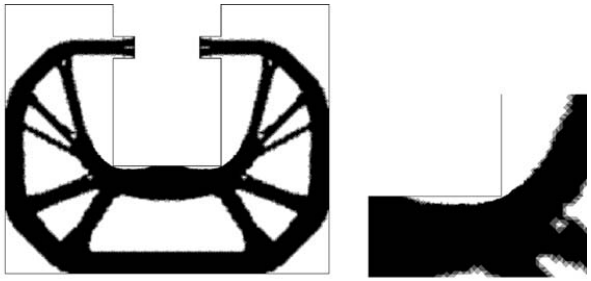
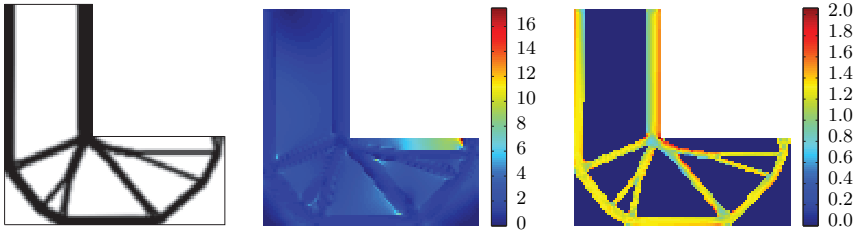
Dealing with the challenge of complexity (2/2)

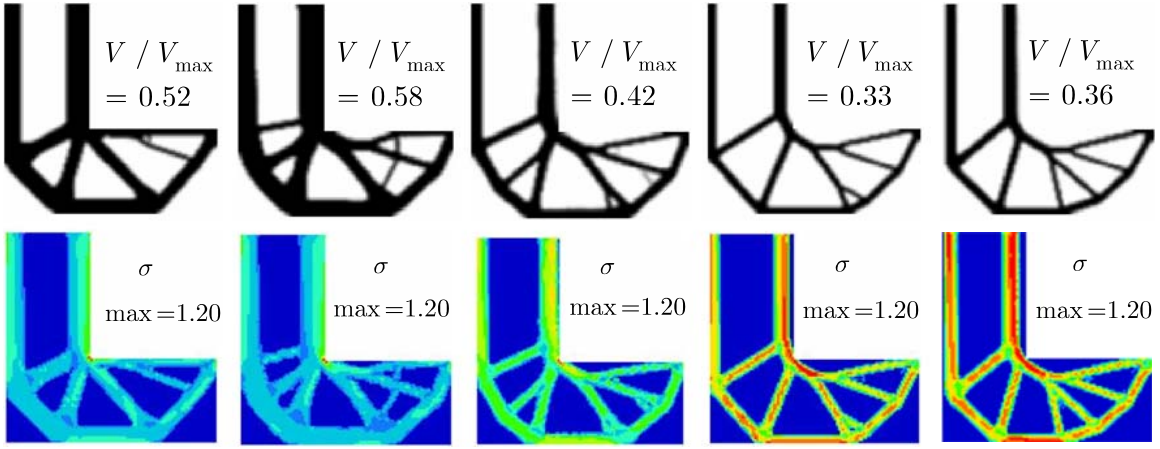


[Amstutz and Novotny, 2010]



(a) Density distribution (b) Unrelaxed stress distribution (c) Stress distribution omitting elements with density $\rho < 1/2$.

[Verbart et al., 2013]



(a) (b) (c) (d) (e)

[Le et al., 2010]

Alternative approach based on elasto-plasticity (1/2)

Find a stress-constrained layout by modeling post-yielding response and driving the design towards "no-yield"

i.e.

Minimize plastic strains s.t. volume and compliance

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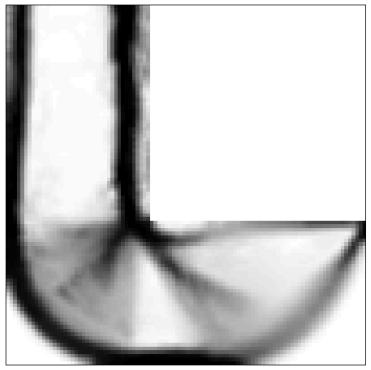
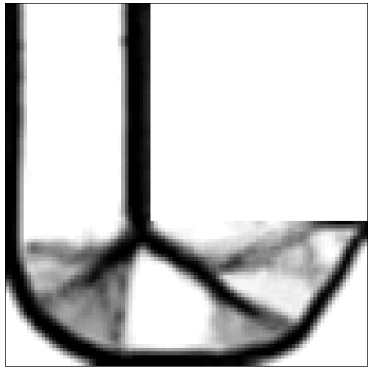
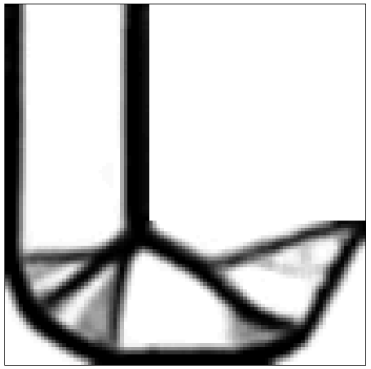


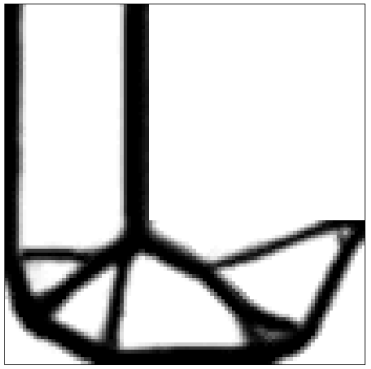
(a) Step 1: 36 design iterations with $p_E = 1.5$,$p_{\sigma_y} = 1.0$, filter radius $r = 0.015$ and hardening $H = 0.001$.



(b) Step 2: 35 further design iterations with $p_E = 2.0$,$p_{\sigma_y} = 1.5$, filter radius $r = 0.015$ and hardening $H = 0.001$.



(c) Step 3: 51 further design iterations with $p_E = 2.5$,$p_{\sigma_y} = 2.0$, filter radius $r = 0.015$ and hardening $H = 0.001$.



(d) Step 4: 35 further design iterations with $p_E = 3.0$,$p_{\sigma_y} = 2.5$, filter radius $r = 0.010$ and hardening $H = 0.01$.

[Amir, 2011]

Alternative approach based on elasto-plasticity (2/2)

Current work:

Minimize volume s.t. plastic strains ($= 0$) and end-compliance

Key aspects:

- Stress constraints are evaluated **accurately** at local material points;
- Formulation involves constraints on **global** quantities only;
- **Nonlinear FE** analysis is required.

Relative complexity: $\uparrow$ analysis $\downarrow$ optimization

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Topology optimization with elasto-plasticity

Optimizing the energy absorbed by the structure
[Yuge and Kikuchi, 1995], [Swan and Kosaka, 1997], [Yuge et al., 1999], [Maute et al., 1998], [Schwarz et al., 2001], [Yoon and Kim, 2007], [Kato et al., 2015]

Crashworthiness design
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Governing equations - elasto-plasticity

Rate-independent plasticity, J_2 flow theory:

von Mises yield criterion:	$f(\boldsymbol{\sigma}, \kappa) = \sqrt{3J_2} - \sigma_y(\kappa) \leq 0$
Bi-linear isotropic hardening:	$\sigma_y(\kappa) = \sigma_y^0 + HE\kappa$
Associative flow rule:	$\dot{\boldsymbol{\epsilon}}^{pl} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}$
Evolution of internal hardening variable:	$\dot{\kappa} = \sqrt{\frac{2}{3}}\left\|\dot{\boldsymbol{\epsilon}}^{pl}\right\|_2$

Solution on a local level by well-known return mapping algorithm [Simo and Taylor, 1986] .

Nonlinear FEA

Recasting as a nonlinear, transient coupled problem [Michaleris et al., 1994] :

$$
\begin{aligned}
{}^{n}\mathbf{R}({}^{n}\mathbf{u}, {}^{n-1}\mathbf{u}, {}^{n}\mathbf{v}, {}^{n-1}\mathbf{v}) &= 0 \\
{}^{n}\mathbf{H}({}^{n}\mathbf{u}, {}^{n-1}\mathbf{u}, {}^{n}\mathbf{v}, {}^{n-1}\mathbf{v}) &= 0
\end{aligned}
$$

$$
{}^{n}\mathbf{v} = \begin{bmatrix} {}^{n}\boldsymbol{\epsilon}^{pl} \\ {}^{n}\kappa \\ {}^{n}\boldsymbol{\sigma} \\ {}^{n}\lambda \end{bmatrix}
$$

Global incremental force equilibrium, displacement control:

$$
{}^{n}\mathbf{R}({}^{n}\mathbf{v}, {}^{n}\theta) = {}^{n}\theta\widehat{\mathbf{f}}_{ext} - \int_V \mathbf{B}^T {}^{n}\boldsymbol{\sigma} dV
$$

Local incremental constitutive equations:

$$
\begin{aligned}
{}^{n}\mathbf{H}_1 &= {}^{n-1}\boldsymbol{\epsilon}^{pl} + ({}^{n}\lambda - {}^{n-1}\lambda)(\frac{\partial f}{\partial {}^{n}\boldsymbol{\sigma}})^T - {}^{n}\boldsymbol{\epsilon}^{pl} \qquad \text{(associative flow)} \\
{}^{n}\mathbf{H}_2 &= {}^{n-1}\kappa + ({}^{n}\lambda - {}^{n-1}\lambda)\sqrt{\frac{2}{3}(\frac{\partial f}{\partial {}^{n}\boldsymbol{\sigma}})^T(\frac{\partial f}{\partial {}^{n}\boldsymbol{\sigma}})} - {}^{n}\kappa \qquad \text{(hardening variable)} \\
{}^{n}\mathbf{H}_3 &= {}^{n-1}\boldsymbol{\sigma} + \mathbf{D}\left[\mathbf{B}{}^{n}\mathbf{u} - \mathbf{B}{}^{n-1}\mathbf{u} - ({}^{n}\boldsymbol{\epsilon}^{pl} - {}^{n-1}\boldsymbol{\epsilon}^{pl})\right] - {}^{n}\boldsymbol{\sigma} \qquad \text{(elastic stress-strain)} \\
{}^{n}\mathbf{H}_4 &= J_2 - \frac{1}{3}(\sigma_y(\kappa))^2 \qquad \text{(yield surface)}
\end{aligned}
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\end{aligned}$$

Problem formulation

$$\min_{\mathbf{x}} f(\mathbf{x}) = \frac{\sum_{i=1}^{N_{elem}} v_i \bar{x}_i}{N_{elem}} \qquad \text{(volume fraction)}$$

$$\text{s.t.:} \qquad g_1(\mathbf{x}) = -{}^N\theta \widehat{\mathbf{f}}_{ext}{}^T {}^N\mathbf{u} + g^\star \leq 0 \qquad \text{(end-compliance, disp. control)}$$

$$g_2(\mathbf{x}) = \sum_{i=1}^{N_{elem}} \sum_{j=1}^{N_{gpts}} {}^N\kappa_{i,j} \leq 0 \qquad \text{(plastic strains)}$$

$$0 \leq x_i \leq 1, \qquad i = 1, ..., N_{dv}$$

$$\text{with:} \qquad \mathbf{R}_n({}^n\mathbf{v}, {}^n\theta) = 0 \qquad n = 1, ..., N$$

$$\mathbf{H}_n({}^n\mathbf{u}, {}^{n-1}\mathbf{u}, {}^n\mathbf{v}, {}^{n-1}\mathbf{v}, \bar{\mathbf{x}}) = 0 \qquad n = 1, ..., N$$

Remarks:

- Physical density $\bar{x}$ from density filter and Heaviside projection [Bruns and Tortorelli, 2001, Bourdin, 2001, Guest et al., 2004, Xu et al., 2010] .
- Solution obtained by MMA [Svanberg, 1987] .

Design parameterization

Modified SIMP for stiffness and yield stress

[Bendsøe, 1989, Sigmund and Torquato, 1997] :

$$E(\bar{x}_i) = E_{min} + (E_{max} - E_{min})\bar{x}_i^{p_E}$$

$$\sigma_y^0(\bar{x}_i) = \sigma_{y,min}^0 + (\sigma_{y,max}^0 - \sigma_{y,min}^0)\bar{x}_i^{p_{\sigma_y}}$$

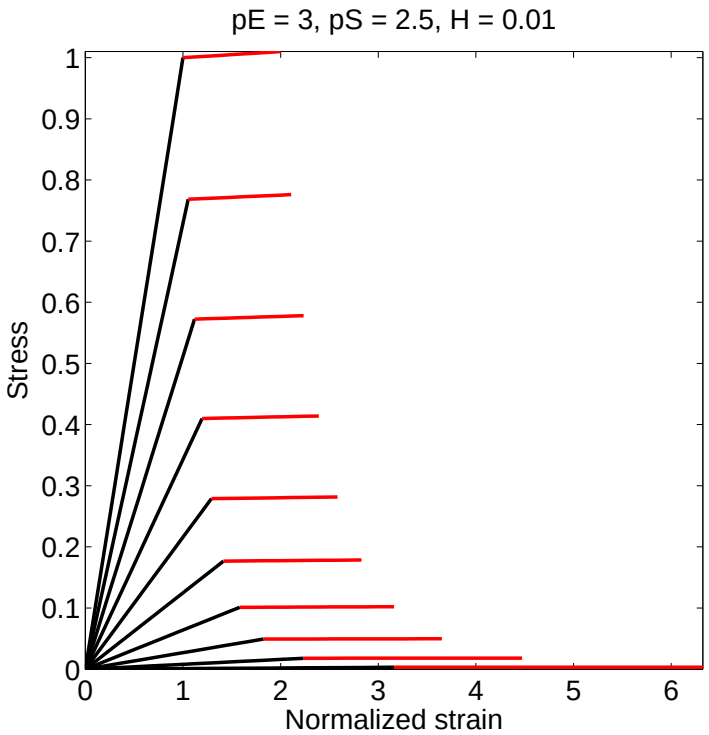
$p_E > p_{\sigma_y}$ [Maute et al., 1998]

↓

"delayed" yield strain for intermediate densities

↓

relaxation of singularity



Adjoint sensitivity analysis

Backwards-incremental adjoint procedure [Michaleris et al., 1994] :

Augmented response functional

$$\widehat{g}(\mathbf{u},\mathbf{v},\theta,\bar{\mathbf{x}}) = g({}^{N}\mathbf{u},{}^{N}\mathbf{v},{}^{N}\theta,\bar{\mathbf{x}}) - \sum_{n=1}^{N} {}^{n}\boldsymbol{\lambda}^{T}\,{}^{n}\mathbf{R}({}^{n}\mathbf{v},{}^{n}\theta) - \sum_{n=1}^{N} {}^{n}\boldsymbol{\gamma}^{T}\,{}^{n}\mathbf{H}({}^{n}\mathbf{u},{}^{n-1}\mathbf{u},{}^{n}\mathbf{v},{}^{n-1}\mathbf{v},\bar{\mathbf{x}})$$

Global adjoint equations for ${}^{n}\boldsymbol{\lambda}$

$$\begin{aligned}
\left[-\frac{\partial({}^{n}\mathbf{R})}{\partial({}^{n}\mathbf{v})}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{v})}^{-1}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{u})}\right]^{T}{}^{n}\boldsymbol{\lambda} &= \frac{\partial g}{\partial({}^{n}\mathbf{u})}^{T} - \left[\frac{\partial g}{\partial({}^{n}\mathbf{v})}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{v})}^{-1}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{u})}\right]^{T} \\
&- \left[\frac{\partial({}^{n+1}\mathbf{H})}{\partial({}^{n}\mathbf{u})} - \frac{\partial({}^{n+1}\mathbf{H})}{\partial({}^{n}\mathbf{v})}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{v})}^{-1}\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{u})}\right]^{T}{}^{n+1}\boldsymbol{\gamma} \\
\frac{\partial({}^{n}\mathbf{R})}{\partial({}^{n}\theta)}^{T}{}^{n}\boldsymbol{\lambda} &= \frac{\partial g}{\partial({}^{n}\theta)}
\end{aligned}$$

Local adjoint equations for ${}^{n}\boldsymbol{\gamma}$

$$\frac{\partial({}^{n}\mathbf{H})}{\partial({}^{n}\mathbf{v})}^{T}{}^{n}\boldsymbol{\gamma} = -\frac{\partial({}^{n}\mathbf{R})}{\partial({}^{n}\mathbf{v})}^{T}{}^{n}\boldsymbol{\lambda} - \frac{\partial({}^{n+1}\mathbf{H})}{\partial({}^{n}\mathbf{v})}^{T}{}^{n+1}\boldsymbol{\gamma} + \frac{\partial g}{\partial({}^{n}\mathbf{v})}^{T}$$

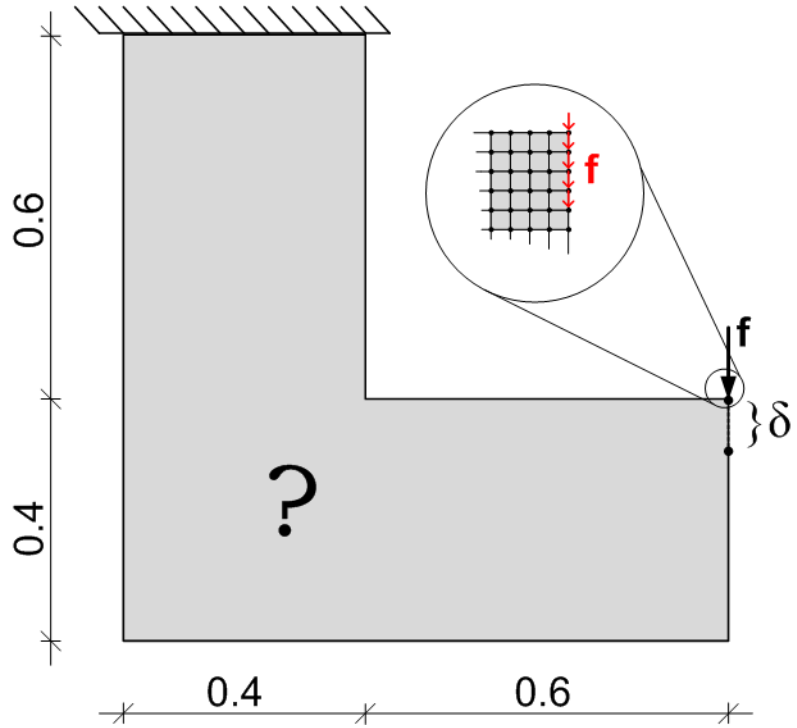
Explicit derivatives w.r.t. design variables

$$\frac{\partial \widehat{g}_{exp}}{\partial \bar{x}_i} = \frac{\partial g}{\partial \bar{x}_i} - \sum_{n=1}^{N} {}^{n}\boldsymbol{\gamma}^{T}\frac{\partial^{n}\mathbf{H}}{\partial \bar{x}_i}$$

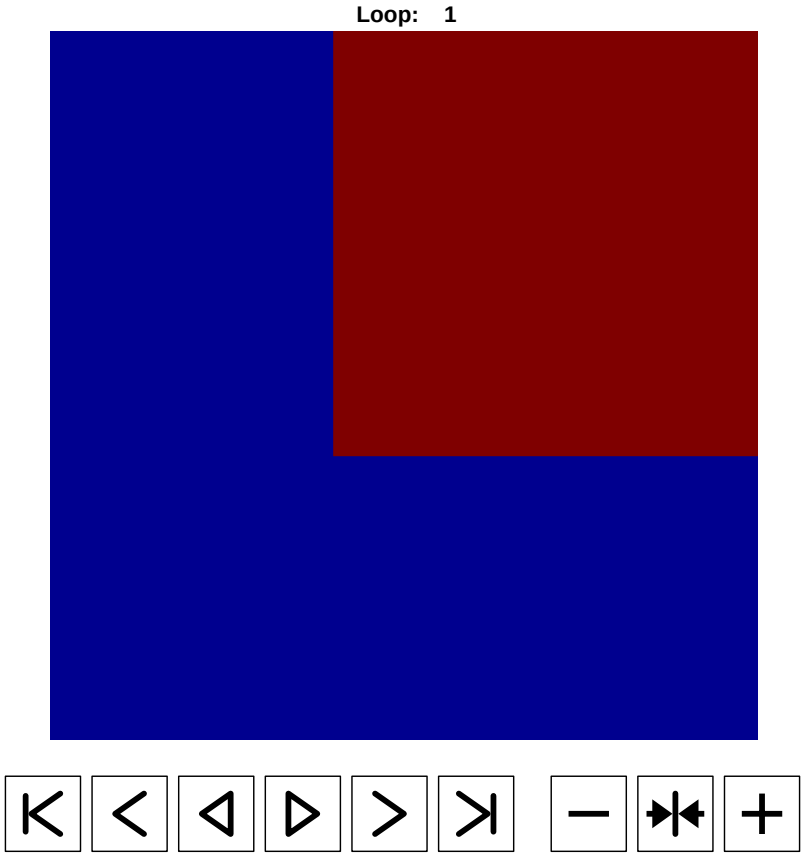
Example: L-bracket (1/3)

Solution parameters:

- nelx = nely = 160, filter radius = 0.025
- $\delta = 0.01$
- $E_{min} = 0.001$, $E_{max} = 1000$, $\sigma^0_{y,min} = 0$, $\sigma^0_{y,max} = 1.8$
- Continuation on p_E, p_{σ_y}, β



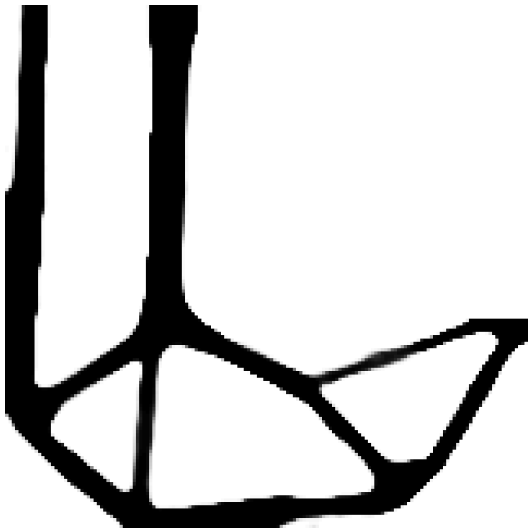
Example: L-bracket (2/3)

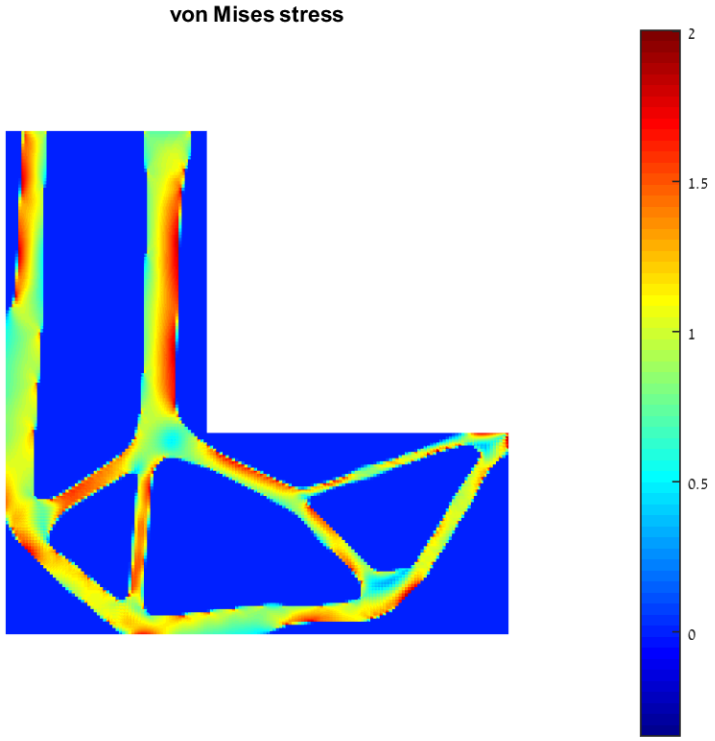


Example: L-bracket (3/3)

In the optimized design:

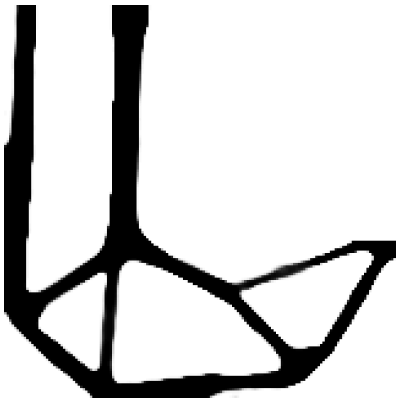
- Reentrant corner is circumvented;
- Compliance constraint is satisfied;
- Maximum stress is the allowable stress.

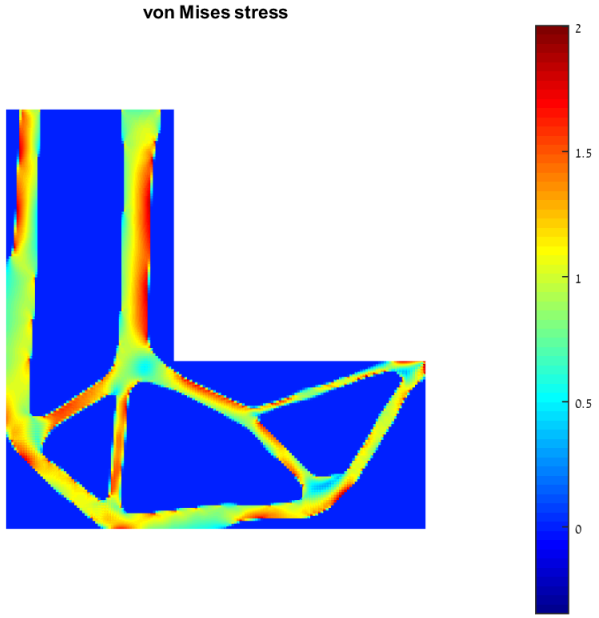




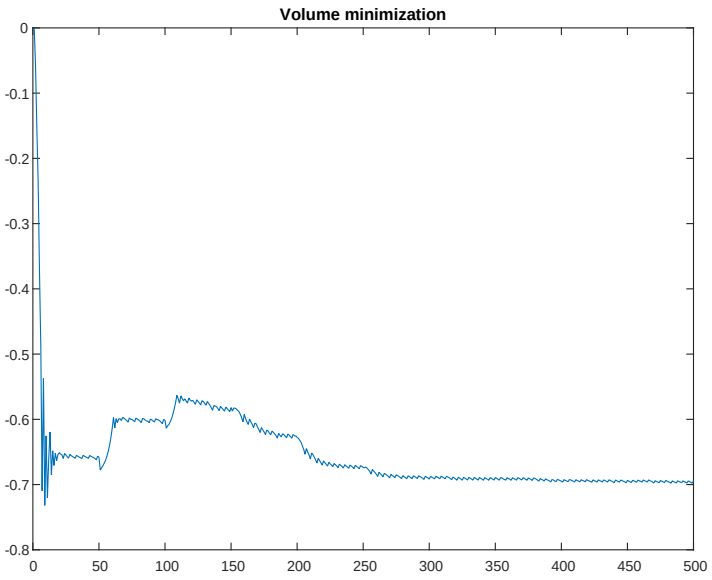
Summary and conclusions

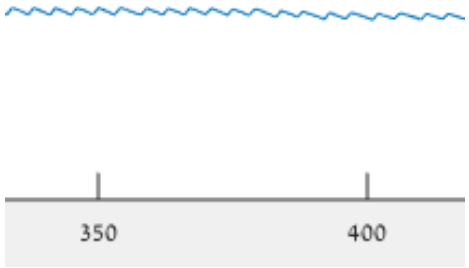
- **Stress constraints** can be achieved via **elasto-plastic modeling** by minimizing or constraining the sum of plastic strains;
- Stress violations are captured **accurately without local constraints**;
- Computational cost **dominated by NLFEA** - can be competitive in large scale;
- Oscillatory behavior - still much room for improvements.



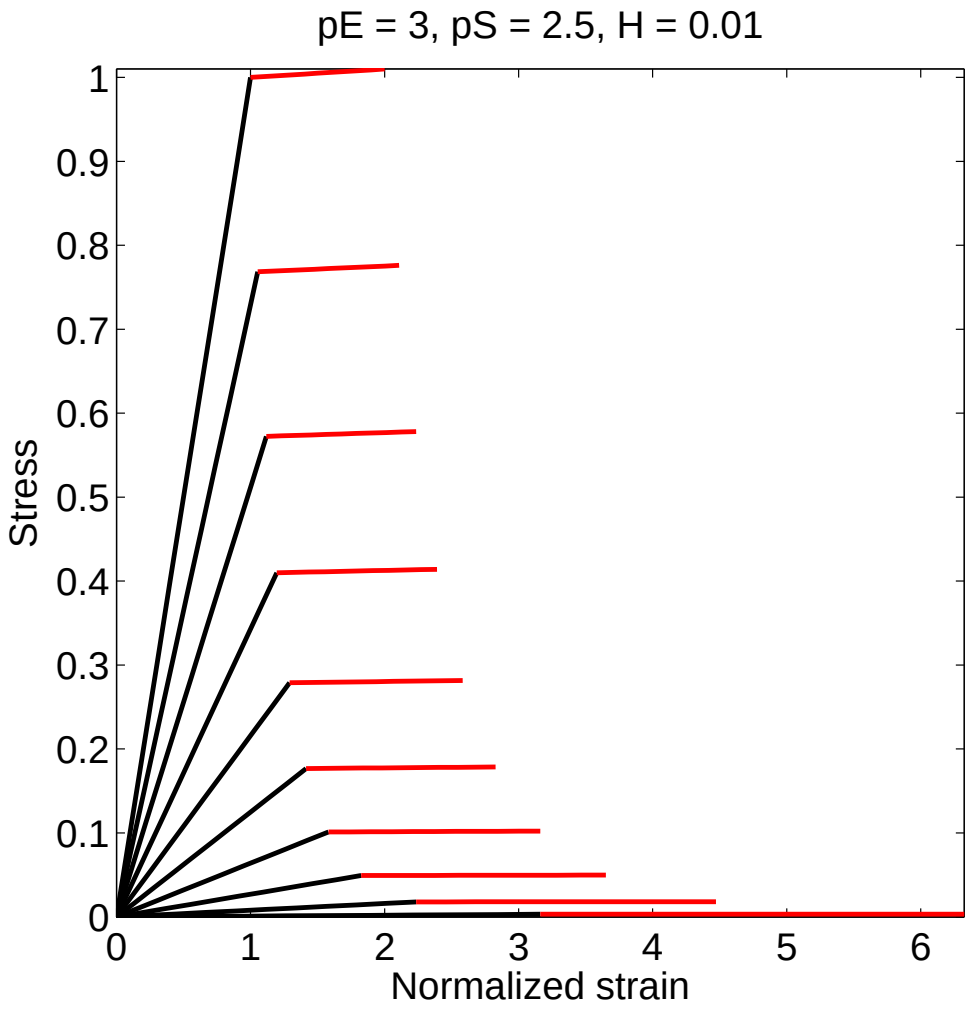


Work in progress - oscillations

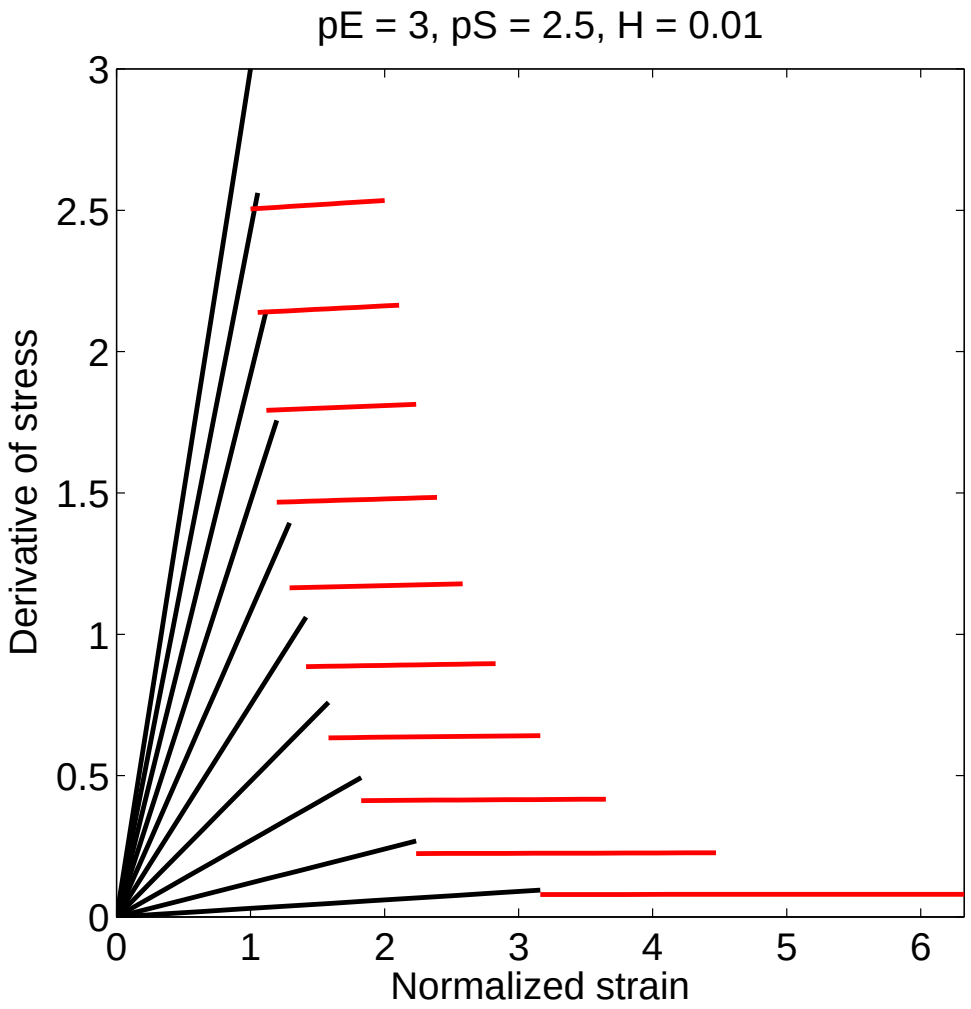




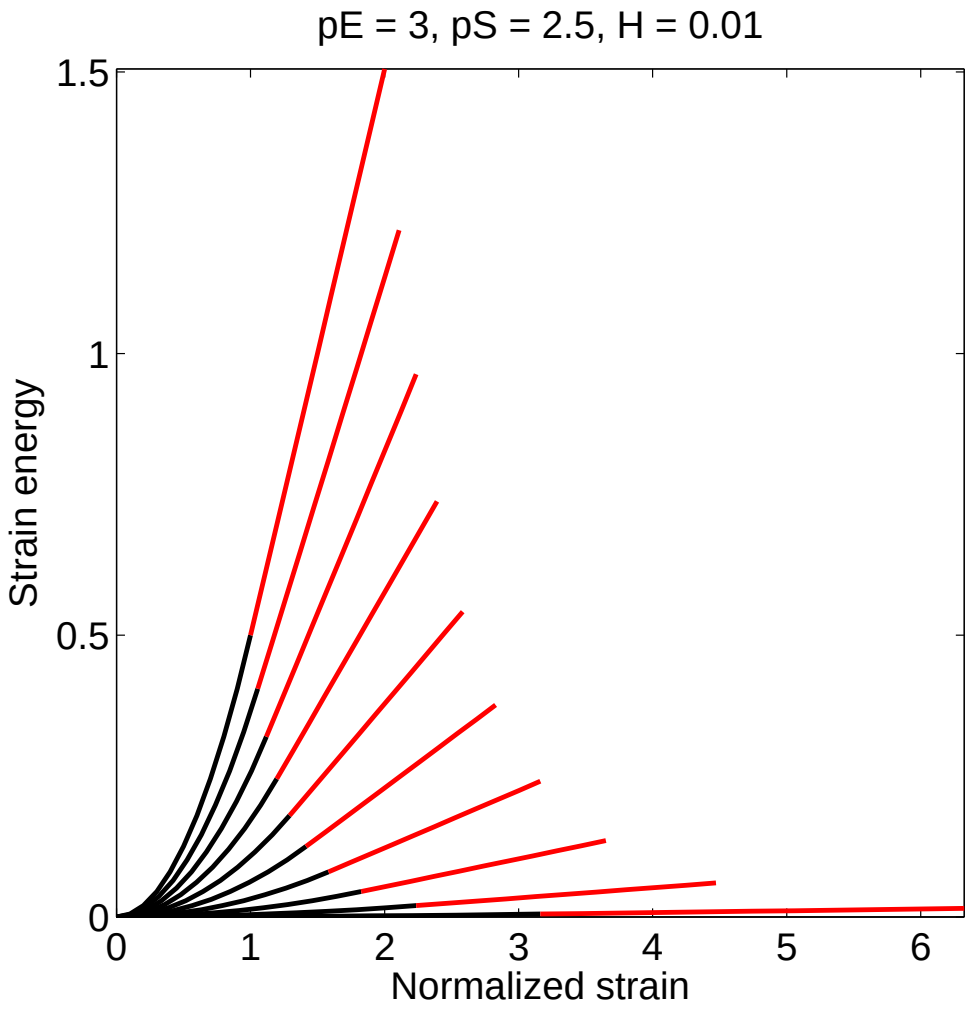
Stress


pE = 3, pS = 2.5, H = 0.01
Stress
Normalized strain
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6

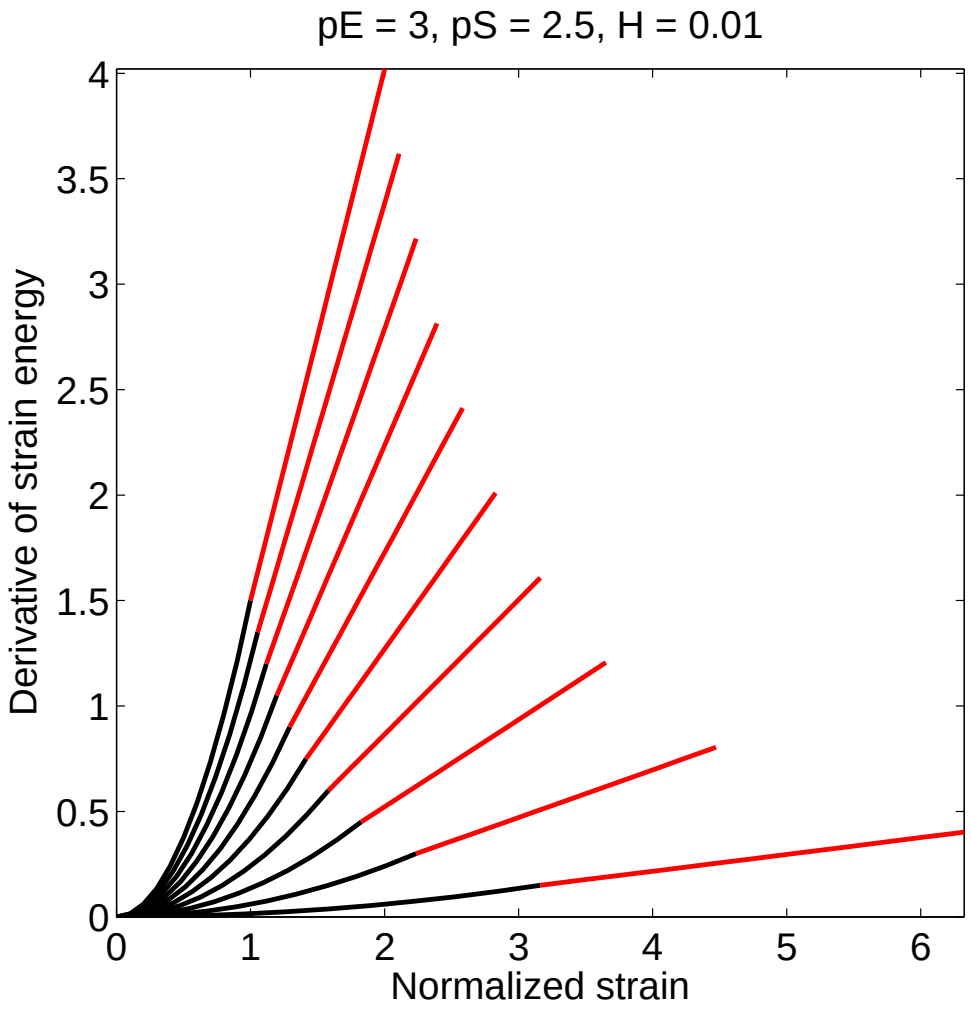
Sensitivity of stress



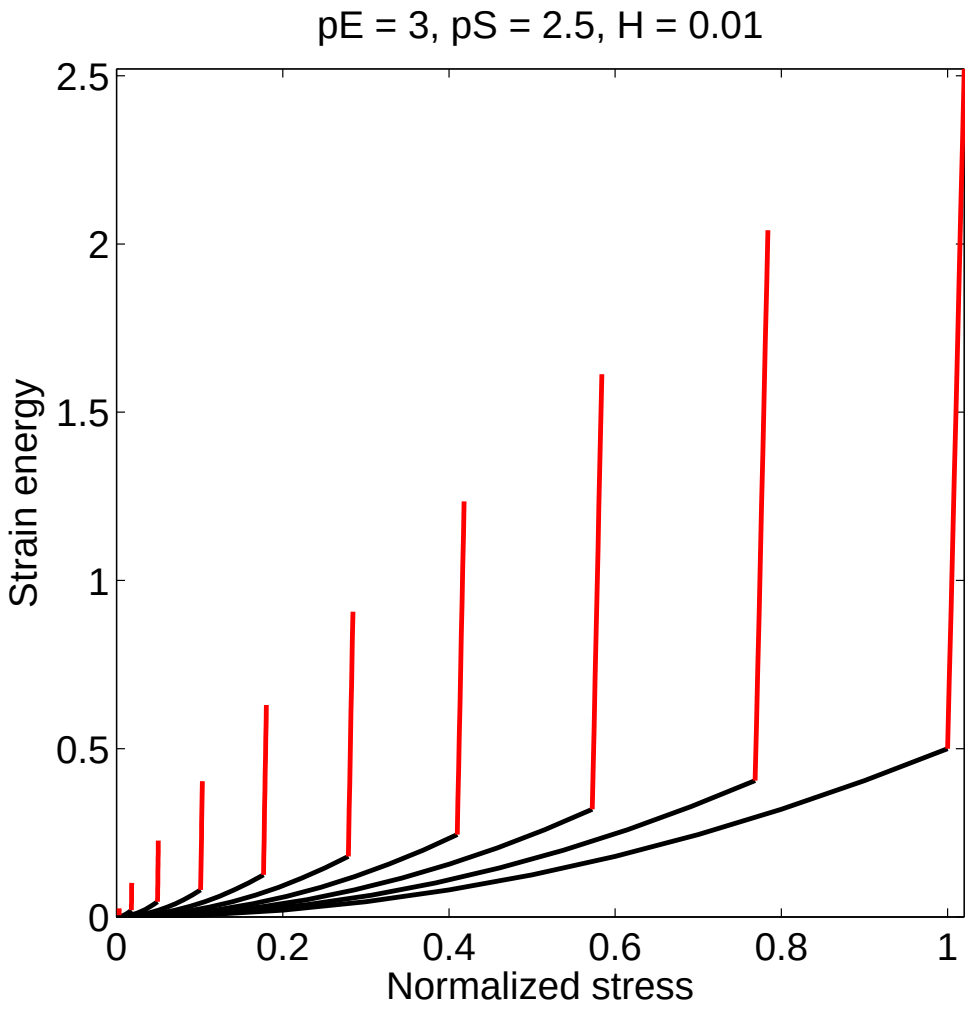
Strain energy vs. strain



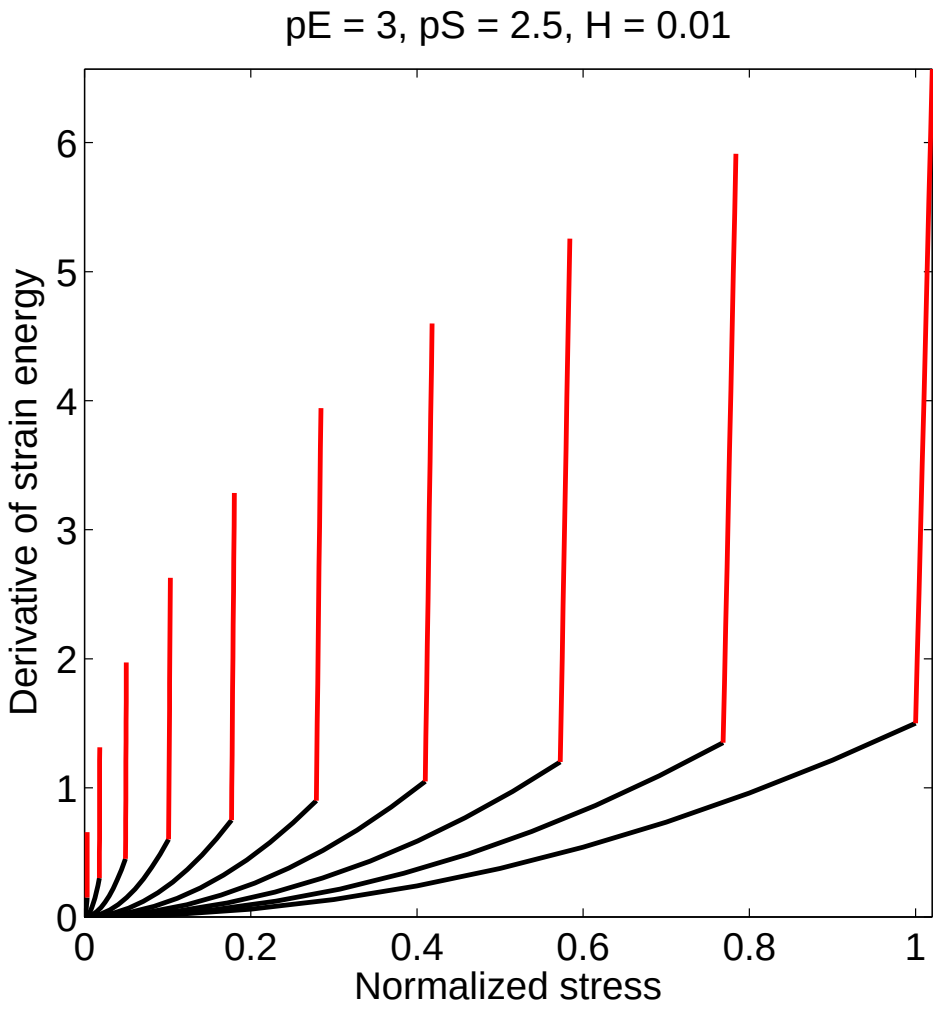
Sensitivity of strain energy vs. strain



Strain energy vs. stress



Sensitivity of strain energy vs. stress



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