Minimum-cost topology and sizing optimization of viscous dampers for seismic retrofitting of 3-D frame structures

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Earthquakes

Devastating natural events which threaten lives, destroy property, and disrupt life-sustaining services and societal functions.

Gorkha earthquake, 2015 (Nepal). Killed $\approx 8,800$, injured $\approx 23,000$. 
Prevention

Conventional seismic design and retrofitting

New buildings
energy dissipation ≡ plastic hinges
damage ≡ costs

Existing buildings
costly & disruption of architectural features

Seismic protection systems

⇒ Fluid viscous dampers

\[ F = c_d \ddot{u} \]
Seismic retrofitting with fluid viscous dampers

Why optimization?

- Optimal dampers’ distribution;
- Optimal dampers’ size;
- Limit the variety of size-groups;
### Optimization approaches

#### Continuous approaches

- [Gluck et al., 1996]
- [Takewaki, 1997]
- [Lavan and Levy, 2006]
- and others...

- Optimal distributions and sizes of dampers;
- **Computational efficient**;
- **Effective** for **large scale** problems;
- Wide variety of damping coefficients.

#### Discrete approaches

- [Zhang and Soong, 1992]
- [Dargush and Sant, 2005]
- [Kanno, 2013]
- and others...

- **Practical distributions** and **sizes** of dampers;
- **Computationally robust**;
- Available dampers’ sizes predefined;
- Computationally expensive for large scale problems.
Mixed-integer approaches

Damper **placement**, **sizing** and **selection**, discrete and continuous variables.

[Lavan and Amir, 2014]: minimum dampers’ cost, inter-story drift constrained, dampers’ distribution, selection, and sizing are variables of the problem, SLP with material interpolation functions;

[Pollini et al., 2014]: minimum realistic retrofitting cost, inter-story drift constrained, dampers’ distribution, selection, and sizing are variables of the problem, GA;

**Observations**

- Most **realistic description** of the problem;
- **Practical** design **solutions**;
- **No aspect** of the design is **pre-defined**;
- Gradient-based: computationally efficient for large-scale problems, but not user-friendly;
- Zero order: computationally expensive for large-scale problems, but more user-friendly.
Goals of the current study

Approach for minimum-cost distributions of fluid viscous dampers combining concepts from *continuum topology* and *discrete material* optimization.

- Realistic retrofitting cost function;
- Formulation with discrete (topology and size selection) and continuous (sizes) variables;
- Final practical solutions with a reasonable computational effort.
Problem formulation

\[
\begin{align*}
\text{min} & \quad \text{COST} \quad \text{- retrofitting} \\
\text{s.t.} & \quad d_i \leq \bar{d} \quad \text{- performance index} \\
\text{with:} & \quad \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_d(\bar{c}_d)]\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{e}_{ag}(t) \\
\mathbf{u}(0) &= 0, \quad \dot{\mathbf{u}}(0) = 0
\end{align*}
\]

Variables:
\begin{itemize}
\item $x_1, x_2$ discrete; $y_1, y_2$ continuous
\item $x_1(1) = 0$ no damper;
\item $x_1(1) = 1, x_2(1) = 0, c_d(1) = y_1\bar{c}_d$;
\item $x_1(1) = 1, x_2(1) = 1, c_d(1) = y_2\bar{c}_d$.
\end{itemize}
New realistic cost function

1. Cost due to the number of bays in which dampers are installed:

\[ J_{\text{bays}} = x_1^T C_{\text{mont}} \]

\[ C_{\text{mont}} = D(C_{m1}) \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} + D(C_{m2}) \begin{bmatrix} 1 + (1.5 + \frac{C_{m1}}{C_{m2}})(1 - x_1(1)) \\ 0 \\ 1 + (1.5 + \frac{C_{m1}}{C_{m2}})(1 - x_1(3)) \\ \vdots \end{bmatrix} \]

2. Cost of the dampers – \( f(\text{peak force \& stroke}) \):

\[ J_{\text{dampers}} = \bar{c}_d x_1^T (y_1 1 + (y_2 - y_1)x_2) \]

3. Cost due to the prototype testing – force-velocity behavior:

\[ J_{\text{sizes}} = C_{\text{type}} \left[ \text{sgn}(x_1^T x_2) + \text{sgn}(x_1^T (1 - x_2)) \right] \]

\[ \Rightarrow J = J_{\text{bays}} + J_{\text{dampers}} + J_{\text{sizes}} \]
Mixed-integer formulation

\[
\min_{x_1, x_2, y} \quad J \quad \text{(cost)}
\]

s. t.:  
\[
d_{c,i} = \max_t(|d_i(t)/d_{all,i}|) \leq 1 \quad \forall t, \forall i = 1, \ldots, N_{drifts} \quad \text{(drifts)}
\]

\[
x_{1,k} = \{0, 1\} \quad k = 1, \ldots, 2N_d \quad \text{(damper existence)}
\]

\[
x_{2,k} = \{0, 1\} \quad k = 1, \ldots, 2N_d \quad \text{(size-group association)}
\]

\[
0 \leq y^L_1 \leq y_1 \leq y^U_1 \leq y^L_2 \quad \text{(1st size-group)}
\]

\[
y^U_1 \leq y^L_2 \leq y_2 \leq y^U_2 \leq 1 \quad \text{(2nd size-group)}
\]

with:  
\[
M\ddot{u}(t) + [C + C_d(\ddot{c}_d)]\dot{u}(t) + Ku(t) = -Me_{ag}(t) \quad \forall t, \forall e_{ag}(t) \in E
\]

\[
u(0) = 0, \quad \dot{u}(0) = 0
\]

[Pollini et al., 2014]
Continuous formulation

\[
\begin{align*}
\text{min } & \quad J \quad \text{(cost)} \\
\text{s. t.: } & \quad d_{c,i} = \max_t \left( |d_i(t)/d_{all,i}| \right) \leq 1 \quad \forall t, \forall i = 1, \ldots, N_{\text{drifts}} \quad \text{(drifts)} \\
& \quad 0 \leq x_{1,k} \leq 1 \quad k = 1, \ldots, 2N_d \quad \text{(existence)} \\
& \quad 0 \leq x_{2,k} \leq 1 \quad k = 1, \ldots, 2N_d \quad \text{(size-group association)} \\
& \quad 0 \leq y_1^L \leq y_1 \leq y_1^U \leq y_2^L \quad \text{(1st size-group)} \\
& \quad y_1^U \leq y_2^L \leq y_2 \leq y_2^U \leq 1 \quad \text{(2nd size-group)} \\
\text{with: } & \quad M\ddot{u}(t) + [C + C_d(\tilde{c}_d)]\dot{u}(t) + K\dot{u}(t) = -M\text{e}_{ag}(t) \forall t, \forall a_{ag}(t) \in \mathcal{E} \\
& \quad u(0) = 0, \quad \dot{u}(0) = 0
\end{align*}
\]
Ingredients

Material interpolation techniques (RAMP, [Stolpe and Svanberg, 2001]) - discrete solutions:

\[
\tilde{c}_{d,j} = \tilde{c}_{d} \left[ \frac{x_{1,j}}{1 + p(1 - x_{1,j})} \right] \left( y_1 + (y_2 - y_1) \left[ \frac{x_{2,j}}{1 + p(1 - x_{2,j})} \right] \right)
\]

Continuously differentiable functions - gradient-based algorithm:

\[
J_{sizes} = C_{type} \left[ \text{sgn}(x_1^T x_2) + \text{sgn}(x_1^T (1 - x_2)) \right] \Rightarrow \\
\tilde{d}_{c,i} = \max_t \left( |d_i(t)/d_{all,i}| \right)
\]

Constraint management - computational cost:

MMA: \( \tilde{d}_{c,active} = \left\{ \tilde{d}_{c,i} | \tilde{d}_{c,i} \geq 0.95 \right\} \)

CPM: \( \tilde{d}_{c,\text{max}} = \frac{1^T D q^{+1} \tilde{d}_c(t_f) 1}{1^T D q \tilde{d}_c(t_f) 1} \)
Notes regarding the algorithm

- Gradient-based algorithm:
  - Sequential convex programming (Method of Moving Asymptotes)
  - Sequential linear programming (Cutting Planes Method)

- Gradients of the constraints from an adjoint sensitivity analysis (additional time-history analyses with final conditions)

- Algorithm sensitive to:
  - Continuation scheme
  - Constraint aggregation/management

- Not yet user-friendly and computationally robust.
Simple case

1 damper size, \( J = J_{\text{dampers}} \), LA02

**DATA OF THE PROBLEM**

\[ \bar{c}_d = 3000 \text{[kN} \cdot \text{s/m]} \]
\[ d_{\text{all}} = 0.009 \text{[m]} \]

**RESULTS**

\[ c_{d1} = 1102.7 \text{ [kN} \cdot \text{s/m]} \]
\[ c_{d2} = 1102.7 \text{ [kN} \cdot \text{s/m]} \]
\[ J = 2203.69 \text{ [kN} \cdot \text{s/m]} \]
Simple case

Convergence to a discrete solution
Simple case

Convergence to a discrete solution
Simple case

Convergence to a discrete solution
Simple case

Convergence to a discrete solution
Example 1

Eight-story asymmetric frame

DATA OF THE STRUCTURE
- columns: 0.5m × 0.5m in frames 1 and 2
- columns: 0.7m × 0.7m in frames 3 and 4
- beams: 0.4m × 0.6m
- floor mass: 0.75[ton/m²]

DATA OF THE PROBLEM
- $\bar{c}_d = 50000[kN\cdot s/m]$
- $d_{all} = 0.035[m]$
- LA16
Example 1 - results

MMA/GA. Locations 1 - 8.

MMA/GA. Locations 9 - 16.

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{J}$, $J$ [kNs/m]</th>
<th>$d_{c,\text{max}}/d_{\text{all}}$</th>
<th>$\tilde{c}_{d1}$ [kNs/m]</th>
<th>$\tilde{c}_{d2}$ [kNs/m]</th>
<th>Func. evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMA</td>
<td>334,656</td>
<td>1.0055 (0.55%)</td>
<td>15,933</td>
<td>31,735</td>
<td>$2923 \approx 10^{3.465}$</td>
</tr>
<tr>
<td>CPM</td>
<td>334,380</td>
<td>1.0072 (0.72%)</td>
<td>15,942</td>
<td>31,638</td>
<td>$2 \cdot 254 \approx 10^{2.706}$</td>
</tr>
<tr>
<td>GA</td>
<td>337,682</td>
<td>1 (0.00%)</td>
<td>15,906</td>
<td>32,490</td>
<td>$20 \cdot 1000 \cdot 110 \approx 10^{6.342}$</td>
</tr>
</tbody>
</table>
Conclusions

- Material interpolation techniques for optimal distribution and sizing of fluid viscous dampers;

- Discrete and practical solutions from a continuum formulation;

- Reasonable computational cost.
Thank you!

Further reading: