Optimal design of skeletal structures with buckling considerations using nonlinear beam modeling

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Structural optimization in civil engineering

- Examples from Skidmore, Owings & Merrill (SOM)

Beghini et al. 2013

Besserud et al. 2013
Structural optimization in civil engineering

- Difficulties with interpreting complicated continuum forms
- Complicated conversion of continuum topology forms to practical construction

Besserud et al. 2013

Mostafavi et al. 2013

Truss optimization with buckling
Utilizing innovative trends

- Robotic construction of optimized trusses
- Optimization based on:
  - Ground structure approach
  - Classical plastic design (LP) formulation

“Practical” design

Design utilizing robotic capabilities
Truss optimization using ground structure approach

- Minimum weight / volume under stress constrains

\[
\min_{a,q} \sum_{i=1}^{n} (a_i l_i)
\]

\[s.t.: \quad Bq = f\]

\[a_i \sigma_{\text{min}} \leq q_i \leq a_i \sigma_{\text{max}}\]

\[a_i \geq 0, \quad i = 1..n\]

- Classical approach doesn’t take into account buckling considerations:
  - Local buckling
  - Global buckling
  - Chain stability
How are buckling considerations imposed in literature?

- **Plastic design problem - sequentially**
  - Euler buckling (constraint on each bar)
  - Chain stability (constraint on each sequence of unbraced connected bars)

- **Semi-definite problem**
  - Global buckling (one stability constraint)
  - Chain stability (by overlapping bars)

- **Eigen-value problem**
  - Global buckling
  - Local stability (constraints)

- **Mixed-integer LP**
  - Local buckling (constraint)
  - Chain stability (overlapping bars)
The aim of the current work

• Account for all buckling considerations in a single formulation

Main idea

• Use geometric nonlinear (GNL) beam formulation
• Optimize the response of the nonlinear structure, instead of imposing constraints
Geometric non-linear analysis

- GNL beam element derived using co-rotational formulation
- Kinematic assumptions:
  - Large displacements
  - Large rotations
  - Small strains

\[
K_{t} = B^T CB + \frac{N}{l_n} ZZ^T + \frac{(M_1 + M_2)}{l_n^2} (RZ^T + ZR^T)
\]

- Computational scheme: Newton-Raphson
  - Displacement control equilibrium
    \[
    R(u, \theta) = \theta \hat{f}_{ext} - f_{int}(u) = 0
    \]
Problem formulation - maxF

- Maximization of load-bearing capacity subject to a volume constraint with displacement control

\[ \min f = -\theta \]

\[ s.t.: \quad g = \sum_{i=1}^{n} (\rho_i a_i) \leq V^* \]

\[ \rho_{\text{min}} \leq \rho_i \leq \rho_{\text{max}} \quad i = 1..n \]

with: \( R(\rho, u, \lambda) = \theta \hat{f}_{\text{ext}} - f_{\text{int}}(\rho, u) = 0 \)
Problem formulation - minV

- Minimization of volume subject to a constraint on load-bearing capacity with displacement control

\[
\min f = V \\
\text{s.t.: } g = \theta \geq \theta^* \\
\rho_{\text{min}} \leq \rho_i \leq \rho_{\text{max}} \quad i = 1..n
\]

with: \( R(\rho, u, \lambda) = \theta f_{\text{ext}} - f_{\text{int}}(\rho, u) = 0 \)
Sensitivity analysis and solution

- Non-linear programming by the Method of Moving Asymptotes (MMA) – gradient based algorithm
- Sensitivity analysis following the adjoint method:

\[ \dot{c}(\theta, \rho, u) = c(\theta, \rho, u) - \lambda^T [\theta f_{\text{ext}} - f_{\text{int}}(\rho, u)] \]

\[
\frac{\partial \dot{c}}{\partial \rho_e} = \left( \frac{\partial c}{\partial u} + \lambda^T \frac{\partial f_{\text{int}}}{\partial u} \right) \frac{\partial u}{\partial \rho_e} + \left( \frac{\partial c}{\partial \theta} - \lambda^T f_{\text{ext}} \right) \frac{\partial \theta}{\partial \rho_e} + \lambda^T \frac{\partial f_{\text{int}}}{\partial \rho_e}
\]

\[
\frac{\partial \dot{c}}{\partial \rho_e} = \lambda^T \frac{\partial f_{\text{int}}}{\partial \rho_e}
\]

\[
\begin{bmatrix}
K_{ff} & K_{fp} \\
K_{f}^f & f_{\text{ext}}^p
\end{bmatrix}
\begin{bmatrix}
\lambda_f \\
\lambda_p
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{\partial c}{\partial u_f} \\
\frac{\partial c}{\partial \theta}
\end{bmatrix}
\]

\[
c(\theta, \rho, u) = -\theta
\]

\[
\begin{bmatrix}
K_{ff} & K_{fp} \\
K_{f}^f & f_{\text{ext}}^p
\end{bmatrix}
\begin{bmatrix}
\lambda_f \\
\lambda_p
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]
Demonstration: the effect of imperfections

Ground structure

Local imperfection

Global imperfection

Prescribed disp.: Small $u_p$

Optimized layout: Local only

Imp.: Local only

Large $u_p$

Local and global

Truss optimization with buckling
Preliminary results

- Global buckling of a cantilever – Ben-Tal et al. 2000

\[ \theta P = 0.2 \]

\[ u_p = 0.2 \]

\[ 8 \times 1.0 = 8.0 \]
Preliminary results

• Local stability of a cantilever – Achtziger 1999

Ongoing work!
Preliminary results

- Double-clamped truss

Ground structure with local imp. and overlaps

Increasing $u_p$

Truss optimization with buckling
Summary

• GNL beam model has been integrated into truss optimization
• Considering buckling through the nonlinear response can replace the imposition of buckling constraints
• Preliminary results resemble those achieved by more traditional approaches involving constraints on local and global stability

Ongoing work

• Choice of local eccentricity – Euler buckling
• Choice of prescribed displacement
• Choice of global imperfection – initial positions of nodes
• Dealing with convergence difficulties for intermediate designs
Thank you!