Topology Optimization Procedures for Reinforced Concrete Design

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What is topology optimization?

Topology optimization is a **computational tool**, enabling us to optimize the **distribution of material** in a given design space.

- Performance can be improved.
- Amount of material used can be reduced.
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A simple example of a beam:

Problem definition: find the stiffer structure, while limiting the volume of material to 50% of the design domain.
Designing a simply-supported beam
Some applications - optimized structures

Topology optimization has been applied in various fields of engineering design: automotive, aircraft, MEMS, nano-optics, electromagnetics, ...

images: Sigmund and Bendsøe 2004; Stromberg et al. 2011; solidThinking Inc. 2011
Digital design & fabrication

Topology optimization of an aerospace part (EADS)
Topology optimization as an architectural design tool
Topology optimization in concrete design

- Most industrial applications: linear-elastic behavior, metallic materials.
- Software: Optistruct (Altair), TOSCA (fe-design), GENESIS (VR&D), plug-in for Abaqus, ...
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Concrete design: established application is in generating strut & tie models:
- Truss-based optimization: Kumar 1978, ... , Ali & White 2001, ...
- Continuum-based optimization: Liang et al. 2000, ... , Gaynor et al. 2012, ...
- Recognized by fib in bulletin 45 - “Practitioners’ guide to finite element modeling of reinforced concrete structures”.

![Concrete Design Diagram]
Optimization in concrete design

From *fib* bulletin 45:

“FEA methods ... can form the basis for design of new, complex, structures that are not easily dimensioned using other rational design methods.”

“In the near future, NLFEA will likely form the main engine in computer-based automated design software, although in a form likely invisible to the user.”
Optimization in concrete design

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Towards simulation-based optimization?

- Advanced constitutive models: Lubliner et al. 1989, Lee & Fenves 1998, Grassl & Jirásek 2006, Červenka et al. 1998, 2008; and many other important contributions: Bažant, deBorst, Feenstra, ...  
- Implemented in: ATENA, Abaqus, ...
Embedded formulation

- Concrete is an isotropic continuum; material described by a gradient enhanced damage model.
- Steel reinforcement consists of 1-D bars; linear elastic behavior.
- Displacements of both phases are compatible using an embedded formulation (Phillips and Zienkiewicz 1976; Chang et al. 1987).

1-D rebar embedded into an isoparametric 2-D element.
Elastic-damage model for concrete

- Model based on “Gradient enhanced damage for quasi-brittle materials” (Peerlings et al. 1996).
- Successfully applied recently for: multiphase material optimization of fiber reinforced composites (Kato et al. 2009); optimization of fiber geometry (Kato and Ramm 2010).

\[
\begin{align*}
\sigma &= (1 - D)C\epsilon \\
D &= D(\kappa) \\
\bar{\epsilon}_{eq} &\geq 0 \\
\dot{\kappa} &\geq 0, \quad \bar{\epsilon}_{eq} - \kappa \leq 0, \quad \dot{\kappa}(\bar{\epsilon}_{eq} - \kappa) = 0 \\
\bar{\epsilon}_{eq} - c\nabla^2\bar{\epsilon}_{eq} &= \epsilon_{eq}
\end{align*}
\]
Elastic-damage model for concrete

Damage law

\[ D = 1 - \frac{\kappa_0}{\kappa} \left( 1 - \alpha + \alpha \exp^{-\beta(\kappa - \kappa_0)} \right) \]

(Mazars and Pijaudier-Cabot 1989)

Equivalent strain

\[ \epsilon_{eq} = \sqrt{3J_2} + ml_1 \]

(Drucker-Prager function)

(example: \( \epsilon_{eq} - \kappa_0 = 0 \))
Elastic-damage model for concrete

**Damage law**

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**Equivalent strain**

\[ \epsilon_{eq} = \frac{1}{1 - \alpha} \left( \sqrt{3J_2} + \alpha I_1 + \beta \langle \epsilon_1 \rangle - \gamma \langle -\epsilon_1 \rangle \right) \]

(Lubliner et al. 1989)

(example: \( \epsilon_{eq} - \kappa_0 = 0 \))
Finite element implementation

Newton-Raphson incremental-iterative equation, disp. control:

\[
\begin{bmatrix}
K^{uu}_{i-1} + K^{bars} & K^{ue}_{i-1} \\
K^{eu}_{i-1} & K^{\epsilon \epsilon}
\end{bmatrix}
\begin{bmatrix}
\delta u_i \\
\delta \epsilon_{eq,i}
\end{bmatrix}
= 
\begin{bmatrix}
\delta \theta \hat{f}^u_{ext} \\
\hat{f}^\epsilon_{i-1}
\end{bmatrix}
- 
\begin{bmatrix}
f^{u}_{int,i-1} + f^{bars}_{int} \\
K^{\epsilon \epsilon} \bar{\epsilon}_{eq,i-1}
\end{bmatrix}
\]

With:

\[
K^{uu}_{i-1} = \int_{\Omega} B^T (1 - D_{i-1}) C B d\Omega
\]

\[
f^{u}_{int,i-1} = \int_{\Omega} B^T \sigma_{i-1} d\Omega
\]

\[
K^{ue}_{i-1} = -\int_{\Omega} B^T C \epsilon_{i-1} q_{i-1} \tilde{N} d\Omega
\]

\[
f^{\epsilon}_{i-1} = \int_{\Omega} \tilde{N}^T \epsilon_{eq,i-1} d\Omega
\]

\[
K^{eu}_{i-1} = -\int_{\Omega} \tilde{N}^T \left( \frac{\partial \epsilon_{eq}}{\partial \epsilon} \right)_{i-1} B d\Omega
\]

\[
q_{i-1} = \begin{cases}
\left( \frac{\partial D}{\partial \kappa} \right)_{i-1} & \tilde{\epsilon}_{eq,i-1} > \kappa_{old} \\
0 & \tilde{\epsilon}_{eq,i-1} \leq \kappa_{old}
\end{cases}
\]

\[
K^{\epsilon \epsilon} = \int_{\Omega} \left( \tilde{N}^T \tilde{N} + B^T c B \right) d\Omega
\]
Problem formulation - concrete & rebar optimization

\[
\min_x \phi(x) = \frac{\sum_{i=1}^{N_{\text{elem}}} \bar{x}_i}{N_{\text{elem}}} \\
\text{(concrete volume fraction)}
\]

s.t.:

\[
g_1(x) = -\theta_{N_{\text{in}}} \hat{f}_p u_{N_{\text{in}}}^p + g^* \leq 0 \quad \text{(end-compliance, pres. DOF)}
\]

\[
g_2(x) = \sum_{i=1}^{N_{\text{bars}}} a_i l_i - \rho V \leq 0 \quad \text{(reinforcement volume fraction)}
\]

\[
0 \leq x_i \leq 1, \quad i = 1, \ldots, (N_{\text{bars}} + N_{\text{elem}})
\]

with:

\[
R_n(u_n, \theta_n, \bar{\varepsilon}_{eq,n}, \kappa_{n-1}, x) = 0 \quad n = 1, \ldots, N_{\text{in}}
\]

\[
H_n(\bar{\varepsilon}_{eq,n}, \kappa_n, \kappa_{n-1}) = 0 \quad n = 1, \ldots, N_{\text{in}} - 1
\]

- Problem solved using MMA (Svanberg 1987) - first order method, sequential convex approximations.
Design parameterization

- Physical density $\bar{x}$ from density filter and Heaviside projection (Bruns and Tortorelli 2001; Bourdin 2001; Guest et al. 2004; Xu et al. 2010)

- Modified SIMP for concrete (Bendsøe 1989, Sigmund and Torquato 1997)

$$E(\bar{x}_i) = E_{min} + (E_{max} - E_{min})\bar{x}_i^{pE}$$

- Linear interpolation for bar areas

$$a_i(x_i) = a_{min} + (a_{max} - a_{min})x_i$$

- Filtering of bar areas according to surrounding concrete

$$\tilde{x}_i = x_i \frac{1}{N_{ij}} \sum_{j \in N_i} (\bar{x}_j)^{pE}$$
Sensitivity analysis

- Derivatives of objective $\phi$ and constraint $g_2$ are straightforward.
- Derivatives of constraint $g_1$ are computed by the adjoint method:

$$
\frac{\partial g_1}{\partial x_i} = - \sum_{n=1}^{N_{in}} \lambda_n^T \frac{\partial R_n}{\partial x_i}
$$

- Backwards-incremental adjoint procedure due to path-dependency (Michaleris et al. 1994):

$$
\tilde{K}_{N_{in}}^T \lambda_{N_{in}} = \left\{ \begin{array}{c}
- \left\{ \frac{\partial g_1}{\partial \bar{u}^T_{N_{in}}} \right\}^T \\
- \left\{ \frac{\partial g_1}{\partial \theta_{N_{in}}} \right\}^T \\
- \left\{ \frac{\partial g_1}{\partial \bar{\epsilon}_{eq,N_{in}}} \right\}^T
\end{array} \right\}
$$

$$
\gamma_{N_{in} - 1} = \frac{\partial g_1}{\partial \kappa_{N_{in} - 1}} - \left\{ \frac{\partial R_{N_{in}}}{\partial \kappa_{N_{in} - 1}} \right\}^T \lambda_{N_{in}}
$$

$$
\tilde{K}_n^T \lambda_n = \left\{ \begin{array}{c}
0 \\
0 \\
\tilde{N}^T \left\{ \frac{\partial H_n}{\partial \bar{\epsilon}_{eq_n}} \gamma_n \right\}
\end{array} \right\}
$$

$$
\gamma_n = - \left\{ \frac{\partial R_{n+1}}{\partial \kappa_n} \right\}^T \lambda_{n+1} - \frac{\partial H_{n+1}}{\partial \kappa_n} \gamma_{n+1}
$$
Deep beam, point load
Result with $\rho = 0.005$, $g^* \approx 0.8 \times g_{\text{rebar only}}$, DP function
Deep beam, point load
Comparing to linear elastic optimization

Damage modeling

With diagonal bars

Without diagonal bars

Linear elastic modeling

Optimized layout

After 100 extra iterations
Deep beam, point load
Effect of ‘yield’ function

$$\epsilon_{eq} = \sqrt{3J_2} + ml_1$$

$$\epsilon_{eq} = \frac{1}{1 - \alpha} \left( \sqrt{3J_2} + \alpha l_1 + \beta \langle \epsilon_1 \rangle - \gamma \langle -\epsilon_1 \rangle \right)$$

$\phi = 0.6564$

$\phi = 0.6714$
Concluding remarks

- Load-bearing per unit weight improved by over 20% for a given range of displacements.
- Nonlinear modeling, and model parameters, make a difference.
- Practical requirements can be taken into account, e.g. clear cover, rebar spacing, minimum reinforcement, allowed deflections etc.

Future outlook: where can we use FEA-based optimization?
- Reduce weight of concrete members.
- Find reinforcement layout in complex structures.
- Optimize retrofitting of existing structures.
- Optimize distribution of multiple materials.
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Further reading: