Topology Optimization Procedures for Reinforced Concrete Design

Oded Amir
odedamir@technion.ac.il

Technion - Israel Institute of Technology
Faculty of Civil & Environmental Engineering

fib Symposium 2013, Tel-Aviv, April 22 2013

What is topology optimization?

Topology optimization is a **computational tool**, enabling us to optimize the **distribution of material** in a given design space.

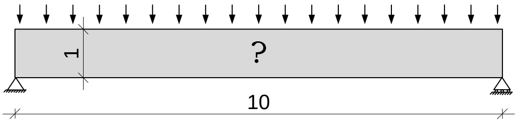
- Performance can be improved.
- Amount of material used can be reduced.

What is topology optimization?

Topology optimization is a **computational tool**, enabling us to optimize the **distribution of material** in a given design space.

- Performance can be improved.
- Amount of material used can be reduced.

A simple example of a beam:

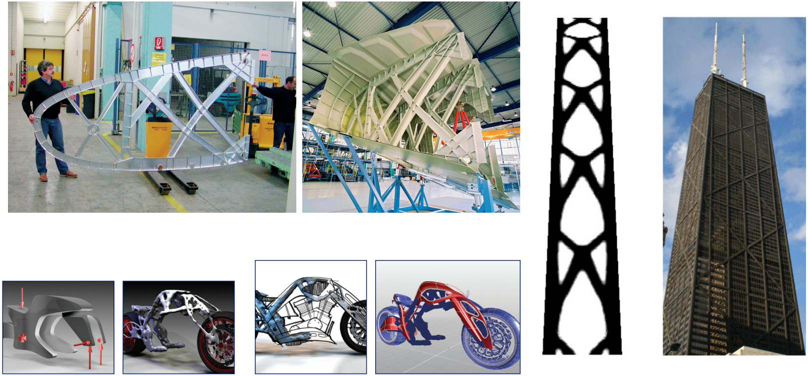


Problem definition: find the **stiffest structure**, while **limiting the volume of material** to 50% of the design domain.

Designing a simply-supported beam

Some applications - optimized structures

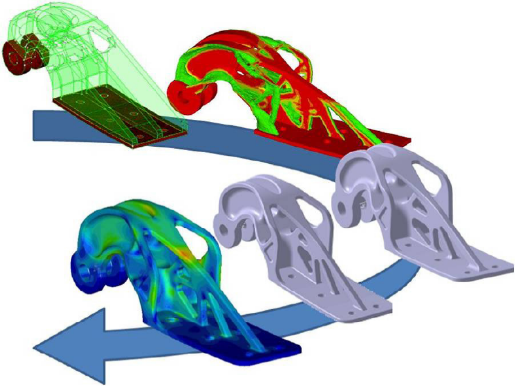
Topology optimization has been applied in various fields of engineering design: automotive, aircraft, MEMS, nano-optics, electromagnetics, ...



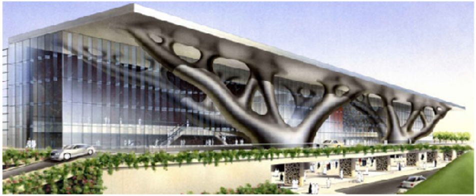
images: Sigmund and Bendsøe 2004; Stromberg *et al.* 2011; solidThinking Inc. 2011

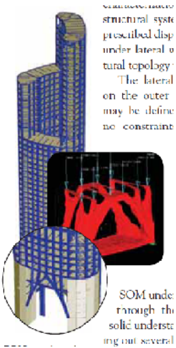
Digital design & fabrication

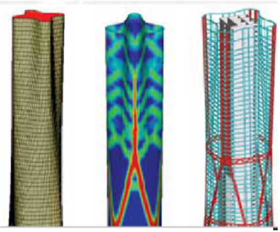
Topology optimization of an aerospace part (EADS)



Topology optimization as an architectural design tool







Topology optimization in concrete design

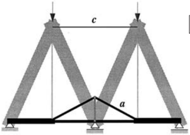
- Most industrial applications: linear-elastic behavior, metallic materials.
- Software: Optistruct (Altair), TOSCA (fe-design), GENESIS (VR&D), plug-in for Abaqus, ...

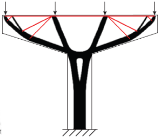
Topology optimization in concrete design

- Most industrial applications: linear-elastic behavior, metallic materials.
- Software: Optistruct (Altair), TOSCA (fe-design), GENESIS (VR&D), plug-in for Abaqus, ...

Concrete design: established application is in generating strut & tie models:

- Truss-based optimization: Kumar 1978, ... , Ali & White 2001, ...
- Continuum-based optimization: Liang et al. 2000, ... , Gaynor et al. 2012, ...
- Recognized by *fib* in bulletin 45 - “Practitioners' guide to finite element modeling of reinforced concrete structures”.





Optimization in concrete design

From *fib* bulletin 45:

"FEA methods ... can form the basis for design of new, complex, structures that are not easily dimensioned using other rational design methods."

"In the near future, NLFEA will likely form the main engine in computer-based automated design software, although in a form likely invisible to the user."

Optimization in concrete design

From *fib* bulletin 45:

"FEA methods ... can form the basis for design of new, complex, structures that are not easily dimensioned using other rational design methods."

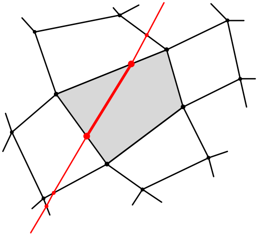
"In the near future, NLFEA will likely form the main engine in computer-based automated design software, although in a form likely invisible to the user."

Towards simulation-based optimization?

- Advanced constitutive models: Lubliner et al. 1989, Lee & Fenves 1998, Grassl & Jirásek 2006, Červenka et al. 1998, 2008; and many other important contributions: Bažant, deBorst, Feenstra, ...
- Implemented in: ATENA, Abaqus, ...

Embedded formulation

- Concrete is an isotropic continuum; material described by a gradient enhanced damage model.
- Steel reinforcement consists of 1-D bars; linear elastic behavior.
- Displacements of both phases are compatible using an embedded formulation (Phillips and Zienkiewicz 1976; Chang et al. 1987) .



1-D rebar embedded into an isoparametric 2-D element

Technion Israel Institute of Technology

Elastic-damage model for concrete

- Model based on "Gradient enhanced damage for quasi-brittle materials" (Peerlings et al. 1996) .
- Successfully applied recently for: multiphase material optimization of fiber reinforced composites (Kato et al. 2009); optimization of fiber geometry (Kato and Ramm 2010).

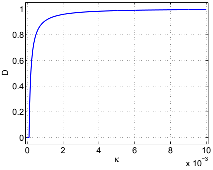
$$
\begin{aligned}
\boldsymbol{\sigma} &= (1-D)\mathbf{C}\epsilon \\
D &= D(\kappa) \\
\bar{\epsilon}_{eq} &\geq 0 \\
\dot{\kappa} \geq 0, \qquad \bar{\epsilon}_{eq} - \kappa &\leq 0, \qquad \dot{\kappa}(\bar{\epsilon}_{eq} - \kappa) = 0 \\
\bar{\epsilon}_{eq} - c\nabla^2\bar{\epsilon}_{eq} &= \epsilon_{eq}
\end{aligned}
$$

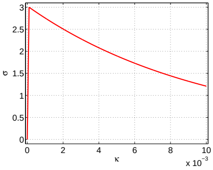
Elastic-damage model for concrete

Damage law

$$D = 1 - \frac{\kappa_0}{\kappa}\left(1 - \alpha + \alpha \exp^{-\beta(\kappa - \kappa_0)}\right)$$

(Mazars and Pijaudier-Cabot 1989)

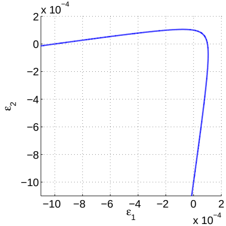




Equivalent strain

$$\epsilon_{eq} = \sqrt{3J_2} + mI_1$$

(Drucker-Prager function)



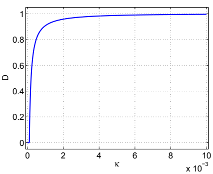
(example: $\epsilon_{eq} - \kappa_0 = 0$)

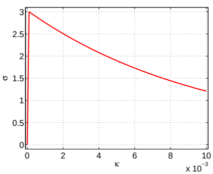
Elastic-damage model for concrete

Damage law

$$D = 1 - \frac{\kappa_0}{\kappa}\left(1 - \alpha + \alpha \exp^{-\beta(\kappa-\kappa_0)}\right)$$

(Mazars and Pijaudier-Cabot 1989)

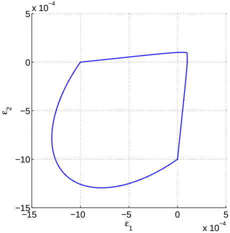




Equivalent strain

$$\epsilon_{eq} = \frac{1}{1-\alpha}\left(\sqrt{3J_2} + \alpha I_1 + \beta\langle\epsilon_1\rangle - \gamma\langle-\epsilon_1\rangle\right)$$

(Lubliner et al. 1989)



(example: $\epsilon_{eq} - \kappa_0 = 0$)

Technion Israel Institute of Technology

Finite element implementation

Newton-Raphson incremental-iterative equation, disp. control :

$$\begin{bmatrix} \mathbf{K}^{uu}_{i-1} + \mathbf{K}^{bars} & \mathbf{K}^{u\epsilon}_{i-1} \\ \mathbf{K}^{\epsilon u}_{i-1} & \mathbf{K}^{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \delta \mathbf{u}_i \\ \delta \bar{\boldsymbol{\epsilon}}_{eq,i} \end{bmatrix} = \begin{bmatrix} \delta\theta \hat{\mathbf{f}}^u_{ext} \\ \mathbf{f}^{\epsilon}_{i-1} \end{bmatrix} - \begin{bmatrix} \mathbf{f}^u_{int,i-1} + \mathbf{f}^{bars}_{int} \\ \mathbf{K}^{\epsilon\epsilon} \bar{\boldsymbol{\epsilon}}_{eq,i-1} \end{bmatrix}$$

With:

$$\mathbf{K}^{uu}_{i-1} = \int_\Omega \mathbf{B}^T (1 - D_{i-1}) \mathbf{C} \mathbf{B} d\Omega \qquad \mathbf{f}^u_{int,i-1} = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_{i-1} d\Omega$$

$$\mathbf{K}^{u\epsilon}_{i-1} = -\int_\Omega \mathbf{B}^T \mathbf{C} \boldsymbol{\epsilon}_{i-1} q_{i-1} \tilde{\mathbf{N}} d\Omega \qquad \mathbf{f}^{\epsilon}_{i-1} = \int_\Omega \tilde{\mathbf{N}}^T \epsilon_{eq,i-1} d\Omega$$

$$\mathbf{K}^{\epsilon u}_{i-1} = -\int_\Omega \tilde{\mathbf{N}}^T \left(\frac{\partial \epsilon_{eq}}{\partial \boldsymbol{\epsilon}} \right)^T_{i-1} \mathbf{B} d\Omega \qquad q_{i-1} = \begin{cases} \left(\frac{\partial D}{\partial \kappa}\right)_{i-1} & \bar{\epsilon}_{eq,i-1} > \kappa_{old} \\ 0 & \bar{\epsilon}_{eq,i-1} \leq \kappa_{old} \end{cases}$$

$$\mathbf{K}^{\epsilon\epsilon} = \int_\Omega \left(\tilde{\mathbf{N}}^T \tilde{\mathbf{N}} + \tilde{\mathbf{B}}^T c \tilde{\mathbf{B}} \right) d\Omega$$

Problem formulation - concrete & rebar optimization

$$\min_{\mathbf{x}} \phi(\mathbf{x}) = \frac{\sum_{i=1}^{N_{elem}} \bar{x}_i}{N_{elem}} \qquad \text{(concrete volume fraction)}$$

s.t.:

$$g_1(\mathbf{x}) = -\theta_{N_{in}} \hat{f}^p u^p_{N_{in}} + g^\star \leq 0 \qquad \text{(end-compliance, pres. DOF)}$$

$$g_2(\mathbf{x}) = \sum_{i=1}^{N_{bars}} a_i l_i - \rho V \leq 0 \qquad \text{(reinforcement volume fraction)}$$

$$0 \leq x_i \leq 1, \qquad i = 1, ..., (N_{bars} + N_{elem})$$

with:

$$\mathbf{R}_n(\mathbf{u}_n, \theta_n, \bar{\epsilon}_{eq,n}, \boldsymbol{\kappa}_{n-1}, \mathbf{x}) = 0 \qquad n = 1, ..., N_{in}$$

$$\mathbf{H}_n(\bar{\epsilon}_{eq,n}, \boldsymbol{\kappa}_n, \boldsymbol{\kappa}_{n-1}) = 0 \qquad n = 1, ..., N_{in} - 1$$

- Problem solved using MMA (Svanberg 1987) - first order method, sequential convex approximations.

Technion Israel Institute of Technology

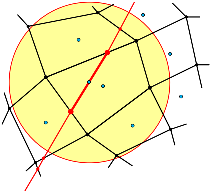
Design parameterization

- Physical density $\bar{x}$ from density filter and Heaviside projection (Bruns and Tortorelli 2001; Bourdin 2001; Guest et al. 2004; Xu et al. 2010)
- Modified SIMP for concrete (Bendsøe 1989, Sigmund and Torquato 1997)

$$E(\bar{x}_i) = E_{min} + (E_{max} - E_{min})\bar{x}_i^{p_E}$$

- Linear interpolation for bar areas

$$a_i(x_i) = a_{min} + (a_{max} - a_{min})x_i$$



- Filtering of bar areas according to surrounding concrete

$$\tilde{x}_i = x_i \frac{1}{N_{ij}} \sum_{j \in N_i} (\bar{x}_j)^{p_E}$$

Sensitivity analysis

- Derivatives of objective ϕ and constraint g_2 are straightforward.
- Derivatives of constraint g_1 are computed by the adjoint method:

$$\frac{\partial g_1}{\partial x_i} = -\sum_{n=1}^{N_{in}} \boldsymbol{\lambda}_n^T \frac{\partial \mathbf{R}_n}{\partial x_i}$$

- Backwards-incremental adjoint procedure due to path-dependency (Michaleris et al. 1994) :

$$\tilde{\mathbf{K}}_{N_{in}}^T \boldsymbol{\lambda}_{N_{in}} = \left\{ \begin{array}{c} -\left\{ \frac{\partial \bar{g}_1}{\partial \mathbf{u}^f_{N_{in}}} \right\}^T \\ \frac{\partial \bar{g}_1}{\partial \theta_{N_{in}}} \\ -\left\{ \frac{\partial \bar{g}_1}{\partial \bar{\epsilon}_{eq,N_{in}}} \right\}^T \end{array} \right\}$$

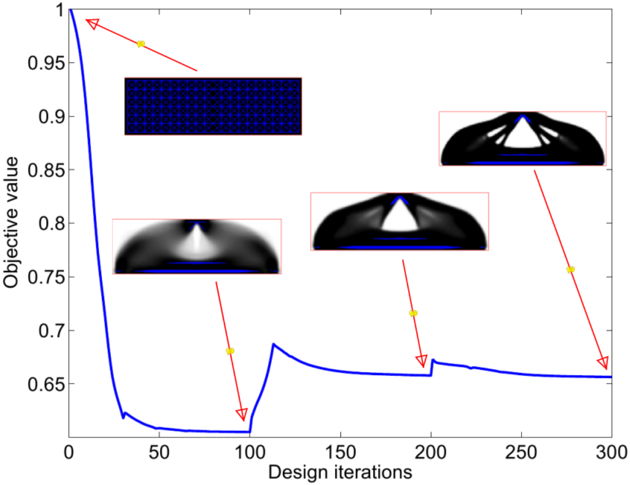
$$\gamma_{N_{in}-1} = \frac{\partial \bar{g}_1}{\partial \kappa_{N_{in}-1}} - \left\{ \frac{\partial \mathbf{R}_{N_{in}}}{\partial \kappa_{N_{in}-1}} \right\}^T \boldsymbol{\lambda}_{N_{in}}$$

$$\tilde{\mathbf{K}}_n^T \boldsymbol{\lambda}_n = \left\{ \begin{array}{c} \mathbf{0} \\ 0 \\ \left\{ \tilde{\mathbf{N}}^T \frac{\partial H_n}{\partial \bar{\epsilon}_{eq,n}} \gamma_n \right\} \end{array} \right\}$$

$$\gamma_n = -\left\{ \frac{\partial \mathbf{R}_{n+1}}{\partial \kappa_n} \right\}^T \boldsymbol{\lambda}_{n+1} - \frac{\partial H_{n+1}}{\partial \kappa_n} \gamma_{n+1}$$

Deep beam, point load

Result with $\rho = 0.005, g^{\star} \approx 0.8 \times g_{rebaronly}$, DP function



Deep beam, point load

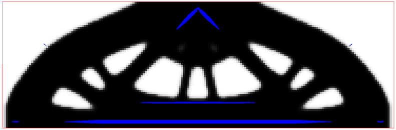
Comparing to linear elastic optimization

Damage modeling

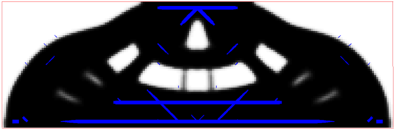
With diagonal bars

Without diagonal bars

Linear elastic modeling



Optimized layout



After 100 extra iterations

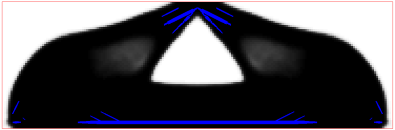
Deep beam, point load

Effect of ‘yield’ function

$$\epsilon_{eq} = \sqrt{3J_2} + mI_1$$

$\phi = 0.6564$

$$\epsilon_{eq} = \frac{1}{1-\alpha}\left(\sqrt{3J_2} + \alpha I_1 + \beta\langle\epsilon_1\rangle - \gamma\langle-\epsilon_1\rangle\right)$$



$\phi = 0.6714$

Concluding remarks

- Load-bearing per unit weight improved by over 20% for a given range of displacements.
- Nonlinear modeling, and model parameters, make a difference.
- Practical requirements can be taken into account, e.g. clear cover, rebar spacing, minimum reinforcement, allowed deflections etc.

Concluding remarks

- Load-bearing per unit weight improved by over 20% for a given range of displacements.
- Nonlinear modeling, and model parameters, make a difference.
- Practical requirements can be taken into account, e.g. clear cover, rebar spacing, minimum reinforcement, allowed deflections etc.

Future outlook: where can we use FEA-based optimization?

- Reduce weight of concrete members.
- Find reinforcement layout in complex structures.
- Optimize retrofitting of existing structures.
- Optimize distribution of multiple materials.

Funding:

Postdoctoral grant from the Danish Council for Independent Research | Technology and Production Sciences.

Further reading:

- Amir O and Sigmund O. Reinforcement layout design for concrete structures based on continuum damage and truss topology optimization. *Structural and Multidisciplinary Optimization* 2013, 47(2):157-174.
- Amir O. A topology optimization procedure for reinforced concrete structures. *Computers and Structures* 2013, 114-115:46-58.