Topology Optimization Procedures for Reinforced Concrete Design

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Motivation

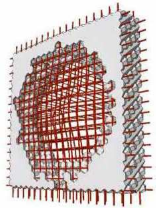
- Concrete is the most consumed man-made material.
- Reinforced concrete uses also 10% of the steel produced.
- Cement production is responsible for 5% of CO_2 emissions.
- Architects are exploring topology optimization as a means of generating aesthetic and efficient structural forms.

Question: Can topology optimization be utilized in the design of efficient concrete structures?

Towards optimizing structural concrete

Challenging characteristics of reinforced concrete:

- Material nonlinearity: quasi-brittle in tension.
- Different behavior in tension and compression.
- Concrete and reinforcement: different physical scales.



→ Established topology optimization procedures are not applicable

BUT: Nonlinear FEA of reinforced concrete has progressed significantly

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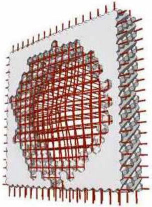

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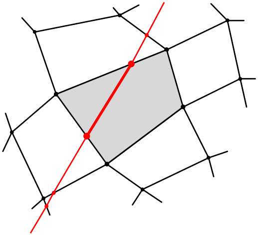
BUT: Nonlinear FEA of reinforced concrete has progressed significantly

The aim: design structural concrete based on topology optimization and nonlinear FEA

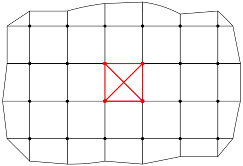
Embedded formulation

- Concrete is an isotropic continuum; material described by a gradient enhanced damage model.
- Steel reinforcement consists of 1-D bars; linear elastic behavior.
- Displacements of both phases are compatible using an embedded formulation (Phillips and Zienkiewicz 1976; Chang et al. 1987) .

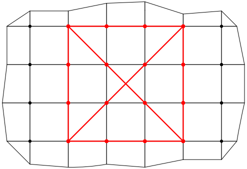


1-D rebar embedded into an isoparametric 2-D element

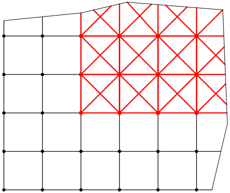
Embedded formulation in topology optimization



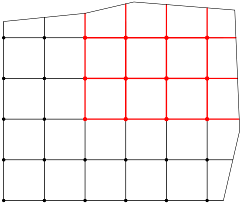
(a) Basic



(b) Large spacing



(c) Clear cover



(d) Without diagonals

Elastic-damage model for concrete

- Model based on "Gradient enhanced damage for quasi-brittle materials" (Peerlings et al. 1996) .
- Successfully applied recently for: multiphase material optimization of fiber reinforced composites (Kato et al. 2009); optimization of fiber geometry (Kato and Ramm 2010).

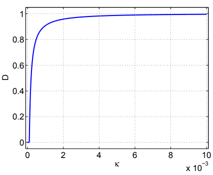
$$
\begin{aligned}
\boldsymbol{\sigma} &= (1-D)\mathbf{C}\boldsymbol{\epsilon} \\
D &= D(\kappa) \\
\bar{\epsilon}_{eq} &\geq 0 \\
\dot{\kappa} \geq 0, \qquad \bar{\epsilon}_{eq} - \kappa &\leq 0, \qquad \dot{\kappa}(\bar{\epsilon}_{eq} - \kappa) = 0 \\
\bar{\epsilon}_{eq} - c\nabla^2\bar{\epsilon}_{eq} &= \epsilon_{eq}
\end{aligned}
$$

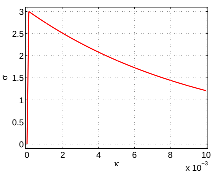
Elastic-damage model for concrete

Damage law

$$D = 1 - \frac{\kappa_0}{\kappa}\left(1 - \alpha + \alpha\,\exp^{-\beta(\kappa-\kappa_0)}\right)$$

(Mazars and Pijaudier-Cabot 1989)

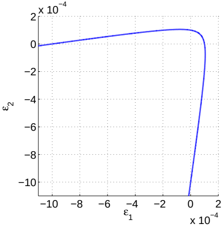




Equivalent strain

$$\epsilon_{eq} = \sqrt{3J_2} + mI_1$$

(Drucker-Prager function)



(example: $\epsilon_{eq} - \kappa_0 = 0$)

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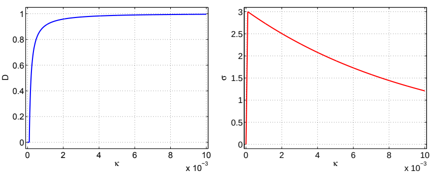
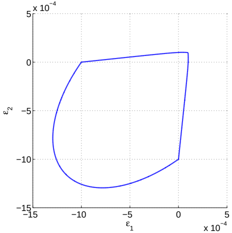


Equivalent strain

$$\epsilon_{eq} = \frac{1}{1-\alpha}\left(\sqrt{3J_2} + \alpha I_1 + \beta\langle\epsilon_1\rangle - \gamma\langle-\epsilon_1\rangle\right)$$

(Lubliner et al. 1989)



(example: $\epsilon_{eq} - \kappa_0 = 0$)

Finite element implementation

Newton-Raphson incremental-iterative equation, disp. control :

$$\begin{bmatrix} \mathbf{K}^{uu}_{i-1} + \mathbf{K}^{bars} & \mathbf{K}^{u\epsilon}_{i-1} \\ \mathbf{K}^{\epsilon u}_{i-1} & \mathbf{K}^{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \delta \mathbf{u}_i \\ \delta \bar{\boldsymbol{\epsilon}}_{eq,i} \end{bmatrix} = \begin{bmatrix} \delta\theta \hat{\mathbf{f}}^u_{ext} \\ \mathbf{f}^{\epsilon}_{i-1} \end{bmatrix} - \begin{bmatrix} \mathbf{f}^u_{int,i-1} + \mathbf{f}^{bars}_{int} \\ \mathbf{K}^{\epsilon\epsilon} \bar{\boldsymbol{\epsilon}}_{eq,i-1} \end{bmatrix}$$

With:

$$\mathbf{K}^{uu}_{i-1} = \int_\Omega \mathbf{B}^T (1 - D_{i-1}) \mathbf{C} \mathbf{B} d\Omega \qquad \mathbf{f}^u_{int,i-1} = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_{i-1} d\Omega$$

$$\mathbf{K}^{u\epsilon}_{i-1} = -\int_\Omega \mathbf{B}^T \mathbf{C} \boldsymbol{\epsilon}_{i-1} q_{i-1} \tilde{\mathbf{N}} d\Omega \qquad \mathbf{f}^{\epsilon}_{i-1} = \int_\Omega \tilde{\mathbf{N}}^T \epsilon_{eq,i-1} d\Omega$$

$$\mathbf{K}^{\epsilon u}_{i-1} = -\int_\Omega \tilde{\mathbf{N}}^T \left(\frac{\partial \epsilon_{eq}}{\partial \boldsymbol{\epsilon}} \right)^T_{i-1} \mathbf{B} d\Omega \qquad q_{i-1} = \begin{cases} \left(\frac{\partial D}{\partial \kappa}\right)_{i-1} & \bar{\epsilon}_{eq,i-1} > \kappa_{old} \\ 0 & \bar{\epsilon}_{eq,i-1} \leq \kappa_{old} \end{cases}$$

$$\mathbf{K}^{\epsilon\epsilon} = \int_\Omega \left(\tilde{\mathbf{N}}^T \tilde{\mathbf{N}} + \tilde{\mathbf{B}}^T c \tilde{\mathbf{B}} \right) d\Omega$$

Problem formulation - concrete & rebar optimization

$$
\begin{aligned}
\min_{\mathbf{x}} \phi(\mathbf{x}) \quad &= \quad \frac{\sum_{i=1}^{N_{elem}} \bar{x}_i}{N_{elem}} && \text{(concrete volume fraction)} \\
\text{s.t.:} \quad & g_1(\mathbf{x}) = -\theta_{N_{in}} \hat{f}^p u^p_{N_{in}} + g^\star \leq 0 && \text{(end-compliance, pres. DOF)} \\
& g_2(\mathbf{x}) = \sum_{i=1}^{N_{bars}} a_i l_i - \rho V \leq 0 && \text{(reinforcement volume fraction)} \\
& 0 \leq x_i \leq 1, \qquad i = 1, ..., (N_{bars} + N_{elem}) \\
\text{with:} \quad & \mathbf{R}_n(\mathbf{u}_n, \theta_n, \bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_{n-1}, \mathbf{x}) = 0 \qquad n = 1, ..., N_{in} \\
& \mathbf{H}_n(\bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_n, \boldsymbol{\kappa}_{n-1}) = 0 \qquad n = 1, ..., N_{in} - 1
\end{aligned}
$$

- Problem solved using MMA (Svanberg 1987) .

Design parameterization

- Physical density $\bar{x}$ from density filter and Heaviside projection (Bruns and Tortorelli 2001; Bourdin 2001; Guest et al. 2004; Xu et al. 2010) .
- Modified SIMP for concrete (Bendsøe 1989, Sigmund and Torquato 1997) :

$$E(\bar{x}_i) = E_{min} + (E_{max} - E_{min})\bar{x}_i^{p_E}$$

- Linear interpolation for bar areas

$$a_i(x_i) = a_{min} + (a_{max} - a_{min})x_i$$

- Filtering of bar areas according to surrounding concrete:

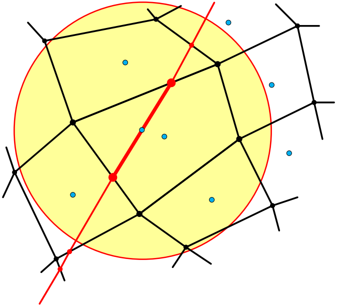
$$\tilde{x}_i = x_i \frac{1}{N_{ij}} \sum_{j \in N_i} (\bar{x}_j)^{p_E}$$

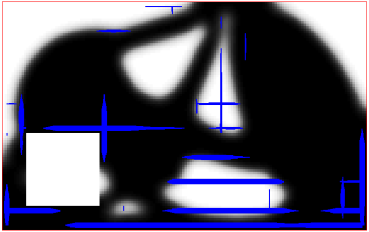
- Mild penalization of intermediate bar stiffnesses:

$$\mathbf{K}_i = E_s(a_{min} + (a_{max} - a_{min})\tilde{x}_i^{p_{bar}})\mathbf{K}_i^0$$

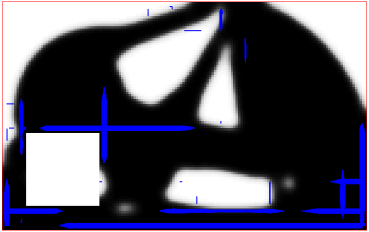
Filtering out ‘floating’ bars

Definition of neighborhood





Without filter



With filter

Sensitivity analysis

- Derivatives of objective ϕ and constraint g_2 are straightforward.
- Derivatives of constraint g_1 are computed by the adjoint method:

$$\frac{\partial g_1}{\partial x_i} = -\sum_{n=1}^{N_{in}} \boldsymbol{\lambda}_n^T \frac{\partial \mathbf{R}_n}{\partial x_i}$$

- Backwards-incremental adjoint procedure due to path-dependency (Michaleris et al. 1994) :

$$\tilde{\mathbf{K}}_{N_{in}}^T \boldsymbol{\lambda}_{N_{in}} = \begin{Bmatrix} -\left\{ \frac{\partial \bar{g}_1}{\partial \mathbf{u}^f_{N_{in}}} \right\}^T \\ \frac{\partial \bar{g}_1}{\partial \theta_{N_{in}}} \\ -\left\{ \frac{\partial \bar{g}_1}{\partial \bar{\epsilon}_{eq,N_{in}}} \right\}^T \end{Bmatrix}$$

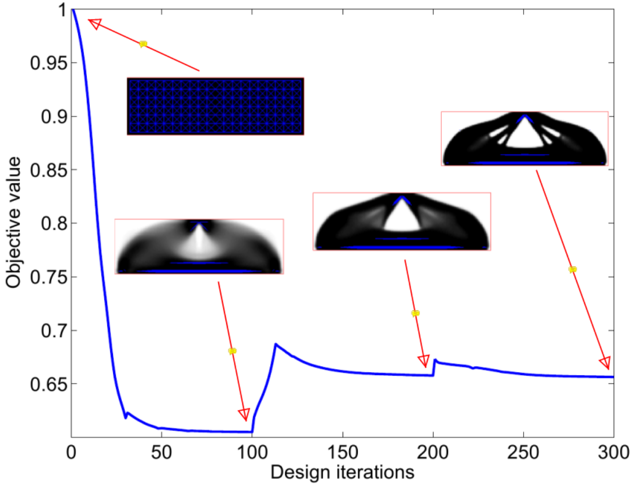
$$\gamma_{N_{in}-1} = \frac{\partial \bar{g}_1}{\partial \kappa_{N_{in}-1}} - \left\{ \frac{\partial \mathbf{R}_{N_{in}}}{\partial \kappa_{N_{in}-1}} \right\}^T \boldsymbol{\lambda}_{N_{in}}$$

$$\tilde{\mathbf{K}}_n^T \boldsymbol{\lambda}_n = \begin{Bmatrix} \mathbf{0} \\ 0 \\ \left\{ \tilde{\mathbf{N}}^T \frac{\partial H_n}{\partial \bar{\epsilon}_{eq,n}} \gamma_n \right\} \end{Bmatrix}$$

$$\gamma_n = -\left\{ \frac{\partial \mathbf{R}_{n+1}}{\partial \kappa_n} \right\}^T \boldsymbol{\lambda}_{n+1} - \frac{\partial H_{n+1}}{\partial \kappa_n} \gamma_{n+1}$$

Deep beam, point load

Result with $\rho = 0.005, g^{\star} \approx 0.8 \times g_{rebaronly}$, DP function



Deep beam, point load

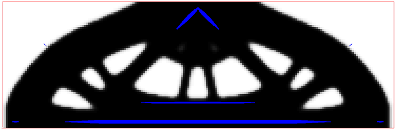
Comparing to linear elastic optimization

Damage modeling

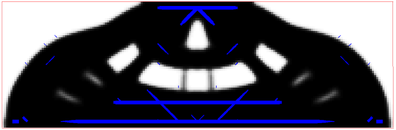
With diagonal bars

Without diagonal bars

Linear elastic modeling



Optimized layout



After 100 extra iterations

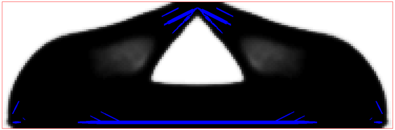
Deep beam, point load

Effect of ‘yield’ function

$$\epsilon_{eq} = \sqrt{3J_2} + mI_1$$

$\phi = 0.6564$

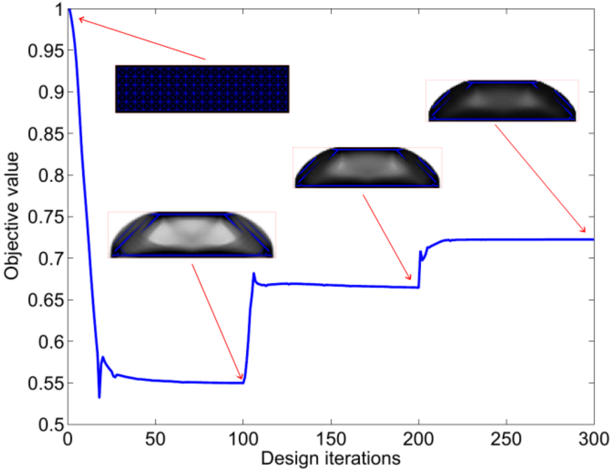
$$\epsilon_{eq} = \frac{1}{1-\alpha}\left(\sqrt{3J_2} + \alpha I_1 + \beta \langle \epsilon_1 \rangle - \gamma \langle -\epsilon_1 \rangle\right)$$



$\phi = 0.6714$

Deep beam, 2 loadcases

Result with $\rho = 0.01, g^{\star} \approx 0.75 \times g_{rebaronly}$, Lubliner function



Concluding remarks

- Some encouraging results: load-bearing capacity per unit weight improved by over 20%.
- Nonlinear modeling, and model parameters, make a difference.
- Practical requirements can be taken into account, e.g. clear cover, rebar spacing, minimum reinforcement, allowed deflections etc.
- Some discouraging results: no holes created, only marginal improvement.

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STILL A LONG WAY TO GO

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Funding:

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Further reading:

- Amir O and Sigmund O. Reinforcement layout design for concrete structures based on continuum damage and truss topology optimization. *Structural and Multidisciplinary Optimization*, online.
- Amir O. A topology optimization procedure for reinforced concrete structures. *Computers and Structures*, in review.