Efficient Computational Procedures for Topology Optimization of Nonlinear Structures

Oded Amir[1], Mathias Stolpe[2] and Ole Sigmund[3]

Technical University of Denmark
[1] PhD Student, Department of Mathematics
[2] Department of Mathematics
[3] Department of Mechanical Engineering

ECCM 2010, Paris, May 17 2010

Introduction - what is this presentation about?

- Examining computational procedures for **topology optimization** of structures that exhibit **nonlinear response**.
- The nested approach is taken -> the **computational bottleneck** is in performing the **nonlinear finite element analysis**.
- Case studies include either **geometric nonlinearities** (large displacements and rotations) or **material nonlinearities** (elasto-plasticity).

Main theme

When performing the nonlinear structural analysis within a certain design cycle, computational effort can be reduced by re-using information corresponding to previous design cycles.

Why consider nonlinear structural response?

Modeling with geometric nonlinearities (GNL):

- Considering **instability** and **buckling**.
- Optimal design is expected to exhibit **large deformations**.
- (e.g. Buhl *et al.* 2000, Pedersen *et al.* 2001, Kemmler *et al.* 2005).

Modeling with material nonlinearities (MNL):

- Maximizing **energy absorption** due to plastic strain (metals).
- **Different strengths** in tension and compression (concrete, rock).
- (e.g. Yuge & Kikuchi 1995, Swan & Kosaka 1997, Maute *et al.* 1998).

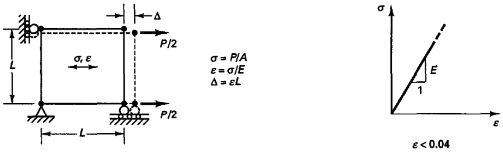
Why consider nonlinear structural response?

Modeling with geometric nonlinearities (GNL):

- Considering **instability** and **buckling**.
- Optimal design is expected to exhibit **large deformations**.
- (e.g. Buhl *et al.* 2000, Pedersen *et al.* 2001, Kemmler *et al.* 2005).

Modeling with material nonlinearities (MNL):

- Maximizing **energy absorption** due to plastic strain (metals).
- **Different strengths** in tension and compression (concrete, rock).
- (e.g. Yuge & Kikuchi 1995, Swan & Kosaka 1997, Maute *et al.* 1998).



(a) Linear elastic (infinitesimal displacements)

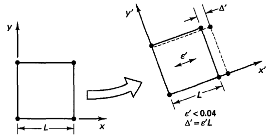
Why consider nonlinear structural response?

Modeling with geometric nonlinearities (GNL):

- Considering **instability** and **buckling**.
- Optimal design is expected to exhibit **large deformations**.
- (e.g. Buhl *et al.* 2000, Pedersen *et al.* 2001, Kemmler *et al.* 2005).

Modeling with material nonlinearities (MNL):

- Maximizing **energy absorption** due to plastic strain (metals).
- **Different strengths** in tension and compression (concrete, rock).
- (e.g. Yuge & Kikuchi 1995, Swan & Kosaka 1997, Maute *et al.* 1998).



(c) Large displacements and large rotations but small strains. Linear or nonlinear material behavior

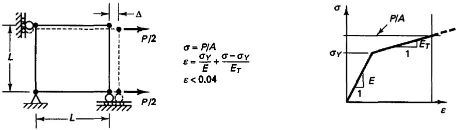
Why consider nonlinear structural response?

Modeling with geometric nonlinearities (GNL):

- Considering **instability** and **buckling**.
- Optimal design is expected to exhibit **large deformations**.
- (e.g. Buhl *et al.* 2000, Pedersen *et al.* 2001, Kemmler *et al.* 2005).

Modeling with material nonlinearities (MNL):

- Maximizing **energy absorption** due to plastic strain (metals).
- **Different strengths** in tension and compression (concrete, rock).
- (e.g. Yuge & Kikuchi 1995, Swan & Kosaka 1997, Maute *et al.* 1998).



(b) Materially-nonlinear-only (infinitesimal displacements, but nonlinear stress-strain relation)

DTU Mathematics
Department of Mathematics

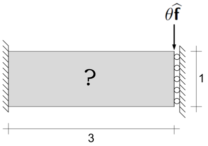
The optimization problem - GNL

$$\min_{\boldsymbol{\rho}} c(\boldsymbol{\rho}) = -\theta\widehat{\mathbf{f}}^T\mathbf{u} \qquad \text{(with prescribed } u_p\text{)}$$

$$\text{s.t.:} \quad \sum_{e=1}^{N} v_e\rho_e \le V \qquad \text{(volume constraint)}$$

$$0 < \rho_{min} \le \rho_e \le 1 \qquad \text{(element densities)}$$

$$\text{with:} \quad \mathbf{R} = \mathbf{f}_{int} - \theta\widehat{\mathbf{f}} = \mathbf{0} \qquad \text{(equilibrium)}$$



* The aim is to **maximize the end-compliance** corresponding to a load $\theta\widehat{\mathbf{f}}$ and a prescribed displacement u_p at a certain DOF.
* Sensitivity analysis requires the solution of an adjoint system: $\frac{\partial \mathbf{f}_{int}}{\partial \mathbf{u}}^T \boldsymbol{\lambda} = -\theta\widehat{\mathbf{f}}$ (with prescribed λ_p).
* $\frac{\partial c}{\partial \rho_e} = -\boldsymbol{\lambda}^T \frac{\partial \mathbf{f}_{int}}{\partial \rho_e}$.

DTU Mathematics
Department of Mathematics

Effect of GNL modeling

Topologies for $V = 0.25 \times V_{total}$

Min. comp., linear modeling

$u_p = 0.005$, GNL

$u_p = 0.2$, GNL

$u_p = 0.5$, GNL

The optimization problem - MNL

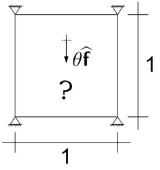
$$\min_{\rho} c(\rho) = -\theta_N \widehat{\mathbf{f}}^T \mathbf{u}_N \qquad \text{(with prescribed } u_p\text{)}$$

$$\text{s.t.:} \quad \sum_{e=1}^{N} v_e \rho_e \leq V \qquad \text{(volume constraint)}$$

$$0 < \rho_{min} \leq \rho_e \leq 1 \qquad \text{(element densities)}$$

$$\text{with:} \quad \mathbf{R}_n = \mathbf{0} \qquad n = 1, ..., N \qquad \text{(path-dependent equilibrium)}$$

$$\mathbf{H}_n = \mathbf{0} \qquad n = 1, ..., N \qquad \text{(local elasto-plastic state)}$$

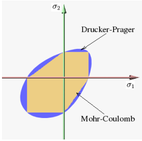


* The aim is to **maximize the end-compliance** corresponding to a load $\theta\widehat{\mathbf{f}}$ and a prescribed displacement u_p at a certain DOF.
* Incremental solution is mandatory due to **path-dependency**.
* Sensitivity analysis involves solving a backwards-incremental, coupled adjoint system (performed following the framework by Michaleris *et al.* 1994).

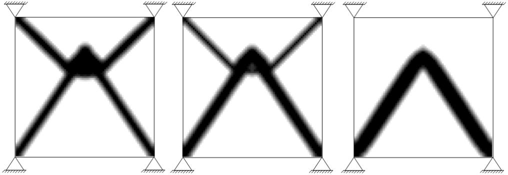
Effect of MNL modeling

Utilizing the **Drucker-Prager yield condition**

$$f(\boldsymbol{\sigma}) = \sqrt{3J_2} + \alpha I_1 - \sigma_y = 0$$



J_2 is the 2nd deviatoric stress invariant;
I_1 is the first stress invariant (trace);
If $\alpha = 0$ -> von-Mises yield criterion.



Topologies for $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 1, 2, 5$
$V = 0.2 \times V_{total}$, $u_p = 0.001$

Examining computational procedures

The focus is on the solution of the nonlinear nested analysis problem using **direct methods**.

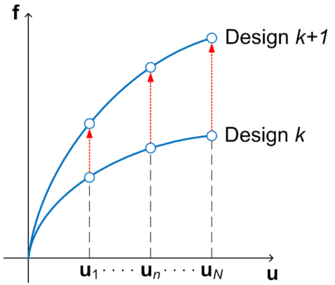
- The problem is linearized using the **Newton-Raphson** iterative procedure.
- Automatic **displacement control** is utilized.
- Incrementation of the displacement is mandatory only for MNL.

Extra cost of sensitivity analysis:

- GNL: Solve adjoint linear system with $\mathbf{K}_T$ corresponding to the converged state, $\mathbf{R} = \mathbf{0}$.
- MNL:
  - Solve multiple linear systems with $\mathbf{K}_T$'s corresponding to the converged states at the end of each increment, $\mathbf{R}_n = \mathbf{0}$.
  - Adjoint load for increment n depends on the solution of the adjoint system for increment $n + 1$.

Re-using information

One possibility is to use **u** and θ corresponding to design cycle k as an **initial guess** for the Newton-Raphson solution within design cycle k+1 -> reduce the number of Newton iterations.



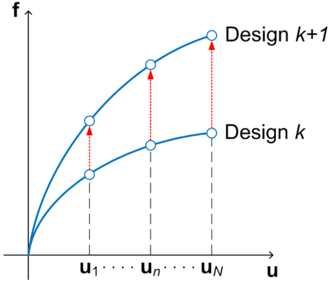
GNL, $u_p = 0.5$, 100 design cycles

Procedure	Total incr.	Newton iter.
Standard	130	1561
Re-use **u**	130	976

DTU Mathematics
Department of Mathematics

Re-using information

One possibility is to use **u** and θ corresponding to design cycle k as an **initial guess** for the Newton-Raphson solution within design cycle k+1 -> reduce the number of Newton iterations.

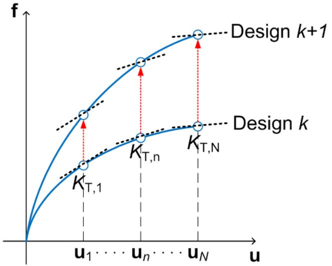


MNL, $u_p = 0.001$, 100 design cycles

Procedure	Total incr.	Newton iter.
Standard, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	200	563
Re-use **u**, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	200	350
Standard, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	200	589
Re-use **u**, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	200	244

Re-using information

Additionally, we can use $\mathbf{K}_T$ corresponding to design cycle k as an approximation of the tangent stiffness in a **Modified Newton-Raphson** solution within design cycle $k+1$ -> reduce the number of matrix factorizations. The re-used $\mathbf{K}_T$'s can be those used in the adjoint solution, where $\mathbf{R} = \mathbf{0}$.

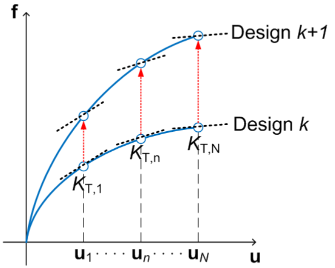


GNL, $u_p = 0.5$, 100 design cycles

Procedure	Newton iter.	Matrix factor.
Standard	1561	1561
Re-use **u** & **K**	1335	814

Re-using information

Additionally, we can use $\mathbf{K}_T$ corresponding to design cycle k as an approximation of the tangent stiffness in a **Modified Newton-Raphson** solution within design cycle $k+1$ -> reduce the number of matrix factorizations. The re-used $\mathbf{K}_T$'s can be those used in the adjoint solution, where $\mathbf{R} = \mathbf{0}$.



MNL, $u_p = 0.001$, 100 design cycles

Procedure	Newton iter.	Matrix factor.
Standard, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	563	563
Re-use $\mathbf{u}$ & $\mathbf{K}$, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	801	187
Standard, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	589	589
Re-use $\mathbf{u}$ & $\mathbf{K}$, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	471	189

Re-using information

The modified Newton approach can be further enhanced in order to reduce the number of Newton iterations and matrix factorizations. Using the same $\mathbf{K}_T$'s, **approximate reanalysis** can be performed, leading to a Newton-Krylov procedure (following Kirsch, Kočvara & Zowe 2002).

Newton iteration

$$\mathbf{K}_T^{k+1}\delta\mathbf{u} = \mathbf{R}$$

Reanalysis equation

$$(\mathbf{K}_T^{k} + \Delta\mathbf{K})\delta\mathbf{u} = \mathbf{R}$$

An approximation to $\delta\mathbf{u}$ is obtained -> not as good as a full Newton step but better than a modified Newton step.

MNL, $u_p = 0.001$, 100 design cycles

Procedure	Newton iter.	Matrix factor.
Re-use **u** & **K**, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	801	187
Reanalysis, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 2$	387	138
Re-use **u** & **K**, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	471	189
Reanalysis, $\frac{\sigma_{y,c}}{\sigma_{y,t}} = 5$	303	177

Summary

DTU Mathematics
Department of Mathematics

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.

DTU Mathematics
Department of Mathematics

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.
- Computational effort can be reduced by re-using information when solving the analysis problem:

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.
- Computational effort can be reduced by re-using information when solving the analysis problem:
    - Solution of the analysis as an initial guess for the analysis in the next design cycle.

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.
- Computational effort can be reduced by re-using information when solving the analysis problem:
  - Solution of the analysis as an initial guess for the analysis in the next design cycle.
  - Factorization of the tangent stiffness matrix as an approximation of the tangent stiffness in a Modified Newton-Raphson procedure in the next design cycle.

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.
- Computational effort can be reduced by re-using information when solving the analysis problem:
  - Solution of the analysis as an initial guess for the analysis in the next design cycle.
  - Factorization of the tangent stiffness matrix as an approximation of the tangent stiffness in a Modified Newton-Raphson procedure in the next design cycle.
- Questions?

Summary

- Structural topology optimization considering geometric or material nonlinearities, following the nested approach.
- Computational effort can be reduced by re-using information when solving the analysis problem:
  - Solution of the analysis as an initial guess for the analysis in the next design cycle.
  - Factorization of the tangent stiffness matrix as an approximation of the tangent stiffness in a Modified Newton-Raphson procedure in the next design cycle.
- Questions?

Thank you for listening!