Topology optimization of post-tensioned concrete beams

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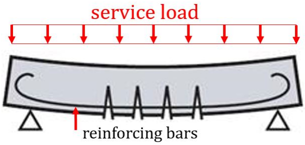
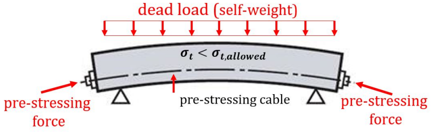
Motivation

Reinforced

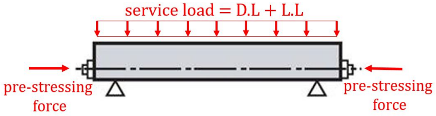


Prestressed



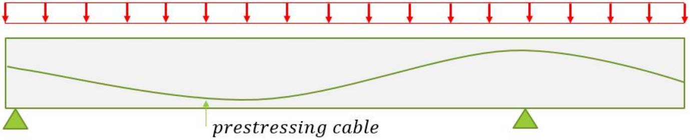
Some advantages of prestressing:

- Durability – no cracks
- Economical – full section is utilized
- Aesthetics – lighter design

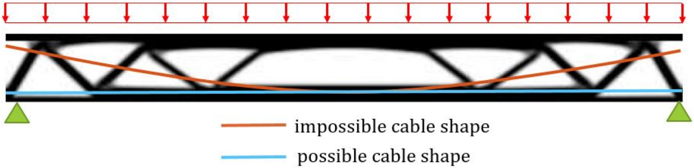


Motivation

Optimizing topology and then adding prestressing doesn't make sense...



cable profile determined according to bending moments



optimized beam did not consider the prestressed tendon

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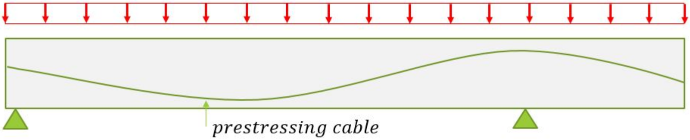

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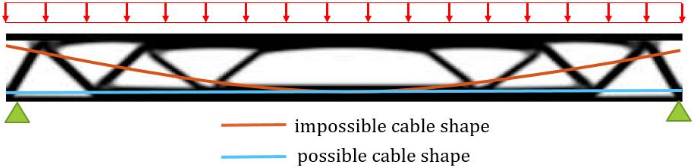
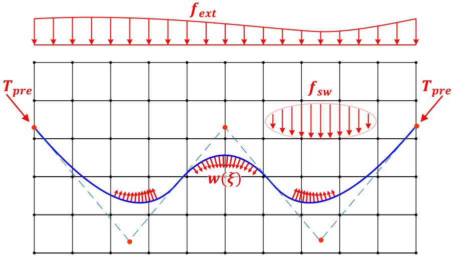


optimized beam did not consider the prestressed tendon

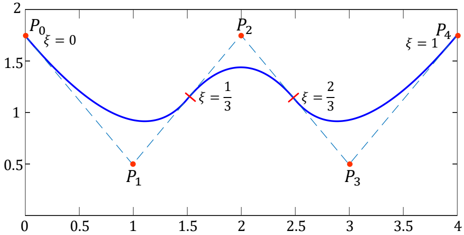
Literature: some work on optimizing tendon profiles; and on TopOpt for given prestressing forces

Modeling approach

- For initial design, prestressing is seen as a collection of forces related to the tendon's curvature



- The tendon geometry is represented by a B-spline
- Control points in X are fixed, coordinates in Y are design variables



Design parametrization

Concrete distribution is determined by filter and projection operations:

1. Density filter

   [Bruns and Tortorelli, 2001, Bourdin, 2001] :

$$\widetilde{\rho}_i = \frac{\sum_{j \in N_i} w(\mathbf{x}_j) v_j \rho_j}{\sum_{j \in N_i} w(\mathbf{x}_j) v_j}$$

2. Tendon-to-concrete filter:

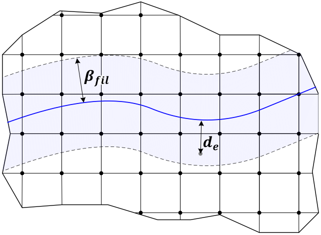
$$\hat{\rho}_i = \widetilde{\rho}_i + (1 - \widetilde{\rho}_i) e^{-\frac{1}{2}\left(\frac{d_i}{\beta_{fil}}\right)^{\mu}}$$

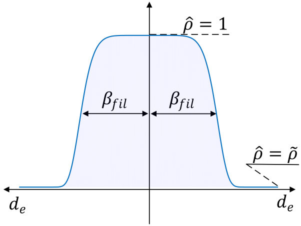
3. Heaviside projections – ‘robust’ approach

   [Guest et al., 2004, Wang et al., 2011, Lazarov et al., 2016] :

$$\overline{\rho}_i^{ero} = \frac{\tanh(\beta_{HS}\eta_{ero}) + \tanh(\beta_{HS}(\hat{\rho}_i - \eta_{ero}))}{\tanh(\beta_{HS}\eta_{ero}) + \tanh(\beta_{HS}(1 - \eta_{ero}))}$$

$$\overline{\rho}_i^{dil} = \frac{\tanh(\beta_{HS}\eta_{dil}) + \tanh(\beta_{HS}(\hat{\rho}_i - \eta_{dil}))}{\tanh(\beta_{HS}\eta_{dil}) + \tanh(\beta_{HS}(1 - \eta_{dil}))}$$





Problem formulation

Minimize **total deformation** and **tendon force**, for a given **volume** of concrete and **allowable curvature**:

$$
\begin{aligned}
\min_{[\boldsymbol{\rho},\mathbf{P},T_{pre}]} \quad & \phi = w_c\left(\mathbf{f}_{ext}^T\mathbf{u}_{total}\right)^2 + w_T\,T_{pre} \\
\text{s.t.:} \quad & g = \frac{\sum_{e=1}^{N_E} \bar{\rho}_e^{dil} v_e}{\sum_{e=1}^{N_E} v_e} - V_{dil}^{\star} \leq 0 \\
& \widetilde{\kappa}_{\max} \leq \bar{\kappa} \\
& 0 \leq \rho_e \leq 1, \qquad e = 1, ..., N_E \\
& \underline{\mathbf{P}} \leq \mathbf{P} \leq \overline{\mathbf{P}} \\
& \mathbf{K}_{ero}\mathbf{u}_{total} = \mathbf{f}_{ext} + \mathbf{f}_{pre}
\end{aligned}
$$

- Density variables $\boldsymbol{\rho}$ and shape variables $\mathbf{P}$ are coupled by the tendon-to-concrete filter

Implementation

- Concrete model is **linear elastic** ☺☺☺

$$E(\overline{\rho}^{ero}) = E_{min} + (E_{max} - E_{min})\,(\overline{\rho}^{ero})^{p_E}$$

- Continuation on parameters:

$$p_E = 1 \to 3 \quad \beta_{HS} = 1 \to 8 \quad \mu = 2 \to 8$$

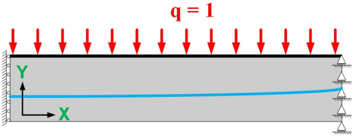
- Heaviside projections with $\eta_{dil} = 0.4$, $\eta_{ero} = 0.6$
- Fixed total of 200 iterations, MMA [Svanberg, 1987] .
- Approximate max. curvature:

$$\kappa(t) = \frac{x(t)'y(t)'' - x(t)''y(t)'}{\left(x(t)'^2 + y(t)'^2\right)^{\frac{3}{2}}} \qquad \widetilde{\kappa}_{\max} = \left(\sum_{k=1}^{N_\kappa} \kappa(t_k)^{p_\kappa}\right)^{\frac{1}{p_\kappa}}$$

Examples

Simply supported, distributed load, 300×30 mesh

- Seek symmetric design
- Either constant or variable tendon force
- No curvature constraint
- 4 control points



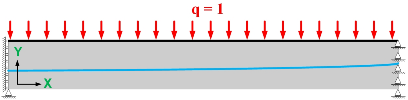
Constant $T_{pre} = 625, \mathbf{f}_{ext}^T \mathbf{u}_{ext} = 223.023, \mathbf{f}_{ext}^T \mathbf{u}_{pre} = -222.305$

Variable $T_{pre} = 627.7, \mathbf{f}_{ext}^T \mathbf{u}_{ext} = 223.810, \mathbf{f}_{ext}^T \mathbf{u}_{pre} = -223.831$

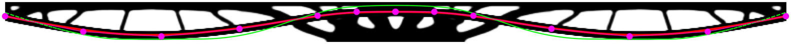
Examples

Continuous beam, distributed load, 400×40 mesh

- With or without curvature constraint
- 7 control points



Constant $T_{pre} = 648.65, \mathbf{f}_{ext}^T\mathbf{u}_{ext} = 362.867, \mathbf{f}_{ext}^T\mathbf{u}_{pre} = -362.863, |\kappa|_{max} = 0.0077$



Constant $T_{pre} = 648.65, \mathbf{f}_{ext}^T\mathbf{u}_{ext} = 414.049, \mathbf{f}_{ext}^T\mathbf{u}_{pre} = -319.919, |\kappa|_{max} = 0.0035$

Constant $T_{pre} = 842.94, \mathbf{f}_{ext}^T\mathbf{u}_{ext} = 429.930, \mathbf{f}_{ext}^T\mathbf{u}_{pre} = -429.932, |\kappa|_{max} = 0.0035$

The effect of T_{pre}

Insight from previous work with piecewise linear tendon

Case			optimized layout
$T_{pre} = 0.6 \times T_{STD}$	ϕ	8.1490e+03	
	$\mathbf{f}_{ext}^T \mathbf{u}_{ext}$	204.6562	
	$\mathbf{f}_{ext}^T \mathbf{u}_{pre}$	-114.3846	
$T_{pre} = 0.8 \times T_{STD}$	ϕ	2.9178e+03	
	$\mathbf{f}_{ext}^T \mathbf{u}_{ext}$	211.6173	
	$\mathbf{f}_{ext}^T \mathbf{u}_{pre}$	-157.6005	
$T_{pre} = 1.0 \times T_{STD}$	ϕ	2.0800e+02	
	$\mathbf{f}_{ext}^T \mathbf{u}_{ext}$	213.1219	
	$\mathbf{f}_{ext}^T \mathbf{u}_{pre}$	-198.6995	
$T_{pre} = 1.2 \times T_{STD}$	ϕ	4.9464e-05	
	$\mathbf{f}_{ext}^T \mathbf{u}_{ext}$	222.6423	
	$\mathbf{f}_{ext}^T \mathbf{u}_{pre}$	-222.6353	
$T_{pre} = 1.4 \times T_{STD}$	ϕ	1.0854e-04	
	$\mathbf{f}_{ext}^T \mathbf{u}_{ext}$	251.9399	
	$\mathbf{f}_{ext}^T \mathbf{u}_{pre}$	-251.9295	

Conclusions and outlook – prestressed concrete

- Coupled shape and topology optimization via **dedicated tendon-to-concrete filter**
- Procedure **captures the essence** of prestressed concrete, leading to no-tension designs
- **Further extensions**: explore objective functionals (stresses?); multiple tendons; ...

Conclusions and outlook – prestressed concrete

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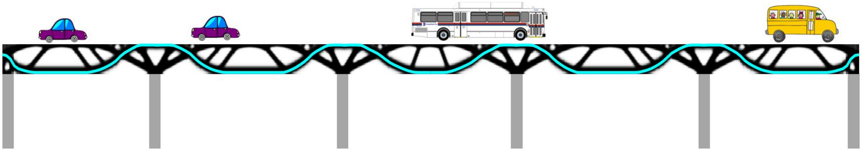
More general view on the approach:

- Related to some ideas on TopOpt with **pressure loads**
- Related to some work on TopOpt with **embedded components**
- Related to emerging **"Geometry Projection"** methods, e.g. Guest, Norato, Guo, ...

Questions?



Further results and discussion:

- Paper in SMO (piecewise linear tendon), 2018
- Paper in current IASS proceedings

References I

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Bruns, T. E. and Tortorelli, D. A. (2001).
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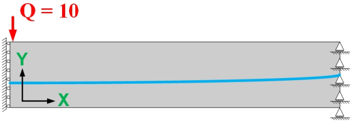
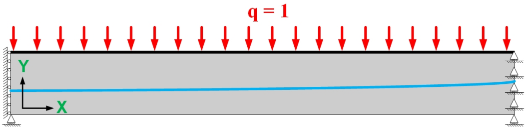
Examples

Further test cases

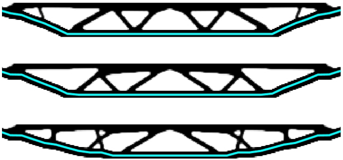
Point load



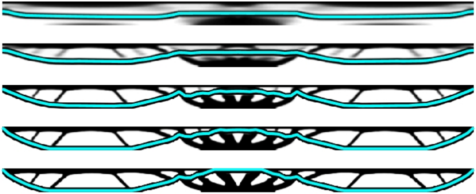
Two-span beam



$T_{pre} = 0.5, 1.0, 1.5 \times T_{STD}$



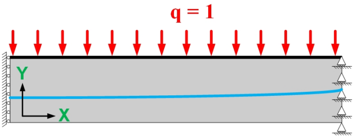
Snapshots with $T_{pre} = 0.8 \times T_{STD}$



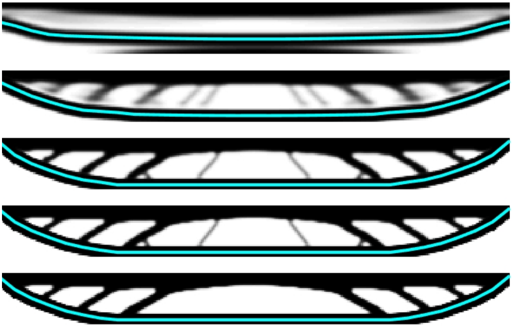
Examples

Further test cases

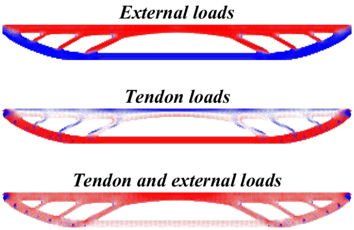
Setup



Optimization snapshots



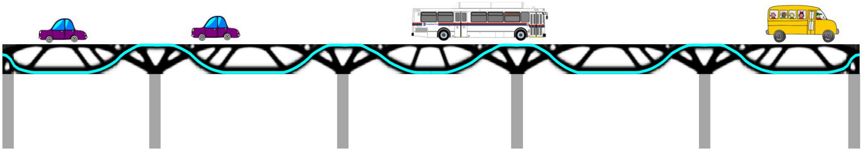
Principal stresses



Examples

Cartoon of a multi-span bridge

Optimizing a multi-span bridge:

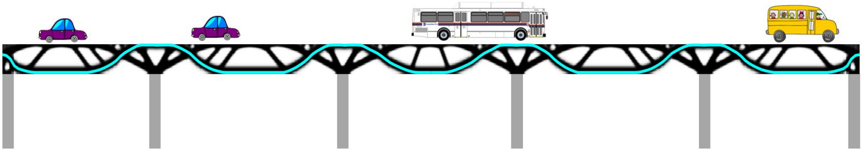


Why doesn't this look "correct"???

Examples

Cartoon of a multi-span bridge

Optimizing a multi-span bridge:



Why doesn't this look "correct"???

Because the construction stages were not considered:

(but that's another talk – see Amir and Mass, SMO 2017, online)

Construction?

- 3-D printing?
- Fabric form?

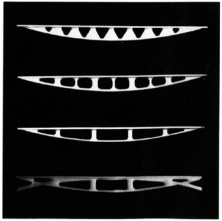


Fig. 5 Some of the many truss designs that can be formed using simple flat, rectangular sheets of fabric as a mold wall.
Abb. 5 Einige der zahlreichen Trägeentwürfe, die mit Schalungen aus einfachen, flachen, rechteckigen Gewebetüchern geformt werden können.

(Thesis of V. Mercuri, Pavia; West, 2006)