Satisfying stress constraints in density based topology optimization by length scale control

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Difficulties with stress constraints

Characteristics of stress-constrained continuum topology optimization:

- Basic engineering requirement: remain linear-elastic, reduce stress concentrations
- Local measure $\rightarrow$ large number of constraints
- Removal of material $\rightarrow$ vanishing of constraint

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Challenge #1: COMPLEXITY

Large number of design variables, large number of constraints

Challenge #2: SINGULARITY

Difficult to capture true optimum by numerical procedures

Successful approaches for constraining stresses

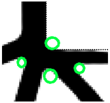
- Consider all local constraints, solve with "active" subsets
- Aggregate local constraints into global stress function, using K-S or p-norm functions
- Apply external penalization on stress violations
- Employ nonlinear modeling or artificial damage

Common to all approaches: the stress / behavior constraint is a function of topological variables

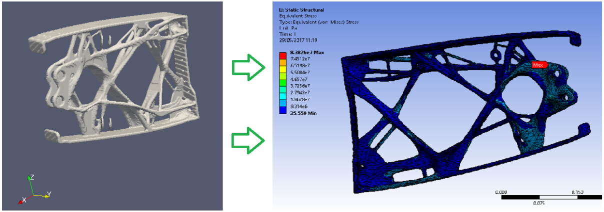
Goal: study the role of length scale

We seek to **study the role of length scale**:

- Stress concentrations / violations are often related to length scale (thickness, curvature):



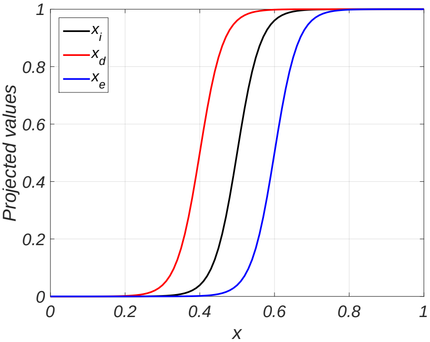
- Shape and sizing following topology may be able to deal with most issues, but creating a parametrized model can be painful:



Controlling the length scale

We follow density-based procedures so control of length scale is via filter radius and Heaviside projections:

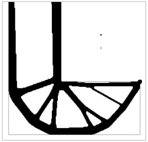
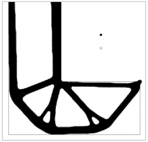
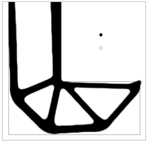
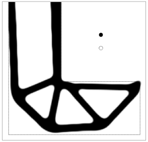
- Well-known density filter (Bruns & Tortorelli 2001, Bourdin 2001)
- "Robust" formulation relying on Heaviside projections (Guest et al. 2004, Sigmund 2009, Wang et al. 2011, Lazarov et al. 2016)



How does length scale influence stresses?

The effect of filter radius, $\eta_d = 0.4$, $\eta_e = 0.6$:

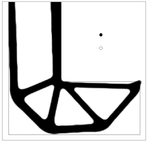
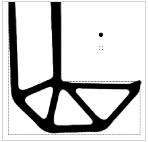
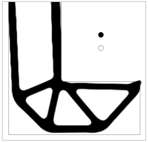
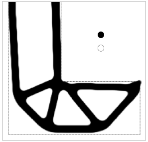
r_{min}, LS $\Uparrow$ compliance $\Uparrow$ stress $\Downarrow$

r_{min}	3	5	7	9
LS	1.9	3.2	4.4	5.7
$\mathbf{f}^T\mathbf{u}$	$2.235 \cdot 10^2$	$2.322 \cdot 10^2$	$2.363 \cdot 10^2$	$2.445 \cdot 10^2$
σ_{VM}^{max}	$6.040 \cdot 10^{-1}$	$5.449 \cdot 10^{-1}$	$4.742 \cdot 10^{-1}$	$4.393 \cdot 10^{-1}$
Layout				

How does length scale influence stresses?

The effect of projection thresholds with r_{min} 7:

LS $\Uparrow$ compliance $\Uparrow$ stress $\Uparrow$

η	$\eta_d = 0.4$ $\eta_e = 0.6$	$\eta_d = 0.3$ $\eta_e = 0.7$	$\eta_d = 0.2$ $\eta_e = 0.8$	$\eta_d = 0.1$ $\eta_e = 0.9$
LS	4.4	6.3	7.7	9.6
$\mathbf{f}^T\mathbf{u}$	$2.363 \cdot 10^2$	$2.403 \cdot 10^2$	$2.454 \cdot 10^2$	$2.531 \cdot 10^2$
σ_{VM}^{max}	$4.742 \cdot 10^{-1}$	$4.765 \cdot 10^{-1}$	$4.879 \cdot 10^{-1}$	$5.055 \cdot 10^{-1}$
Layout				

How does length scale influence stresses?

The effect of filter radius, $\eta_d = 0.4$, $\eta_e = 0.6$:

$r_{min} \Uparrow$ compliance $\Uparrow$ stress $\Uparrow$

r_{min}	3	5	7	9
LS	1.9	3.2	4.4	5.7
$\mathbf{f}^T\mathbf{u}$	$1.633 \cdot 10^2$	$1.702 \cdot 10^2$	$1.725 \cdot 10^2$	$1.818 \cdot 10^2$
σ_{VM}^{max}	$8.186 \cdot 10^{-1}$	$8.307 \cdot 10^{-1}$	$8.388 \cdot 10^{-1}$	$8.591 \cdot 10^{-1}$
Layout				

The filter radius as a design variable

- The filter radius is treated as a **design variable**
- The maximum stress is treated as a **function of the filter radius**

The filter radius as a design variable

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Minimum compliance optimization in two nested loops:

Set initial filter radius, then repeat:

1. Standard minimum compliance (inner loop)
2. Evaluate: $\frac{d\sigma_{max}}{dr_{min}}$
3. Update: $r_{min}^{k+1} = r_{min}^{k} + \frac{\sigma^{\star}_{max} - \sigma_{max}(r^{k}_{min})}{\frac{d\sigma_{max}}{dr_{min}}}$

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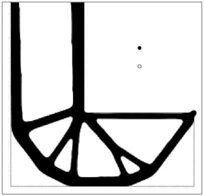
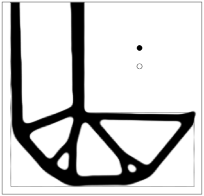
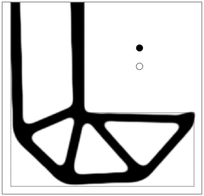
Relying on Bendsøe, Diaz and Kikuchi, 1993:

$$\frac{1}{E}\frac{2(1+\nu)}{3}\boldsymbol{\sigma}^T M\boldsymbol{\sigma} \leq \boldsymbol{\sigma}^T C\boldsymbol{\sigma} \leq \frac{1}{E}2(1-\nu)\boldsymbol{\sigma}^T M\boldsymbol{\sigma}$$

Adaptive filter radius

Initial r_{min} / LS	3.00 / 1.90	5.00 / 3.16	7.00 / 4.43
Final r_{min} / LS	5.69 / 3.60	8.86 / 5.60	11.28 / 7.13
$\sigma^{\star}_{max}$	$5.000 \cdot 10^{-1}$	$4.500 \cdot 10^{-1}$	$4.000 \cdot 10^{-1}$
σ^{max}_{VM}	$5.069 \cdot 10^{-1}$	$4.472 \cdot 10^{-1}$	$4.142 \cdot 10^{-1}$
$\mathbf{f}^T\mathbf{u}$	$2.291 \cdot 10^{2}$	$2.372 \cdot 10^{2}$	$2.434 \cdot 10^{2}$
Layout			

Adaptive filter radius

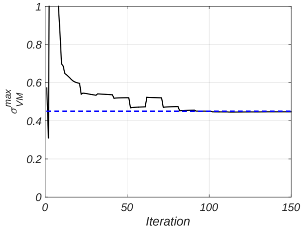
The adaptive filter radius can give superior combinations of compliance and max. stress:

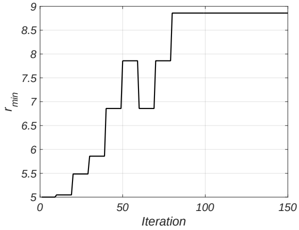
Adaptive $r_{min} = 5.69$	**Constant** $r_{min} = 5.00$
$\mathbf{f}^T\mathbf{u} = 2.291 \cdot 10^2$	$\mathbf{f}^T\mathbf{u} = 2.322 \cdot 10^2$
$\sigma_{VM}^{max} = 5.069 \cdot 10^{-1}$	$\sigma_{VM}^{max} = 5.449 \cdot 10^{-1}$

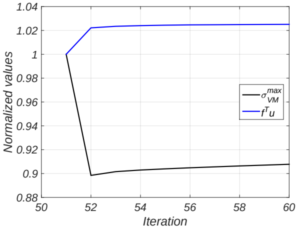
Adaptive $r_{min} = 8.86$	**Constant** $r_{min} = 7.00$
$\mathbf{f}^T\mathbf{u} = 2.372 \cdot 10^2$	$\mathbf{f}^T\mathbf{u} = 2.363 \cdot 10^2$
$\sigma_{VM}^{max} = 4.472 \cdot 10^{-1}$	$\sigma_{VM}^{max} = 4.742 \cdot 10^{-1}$

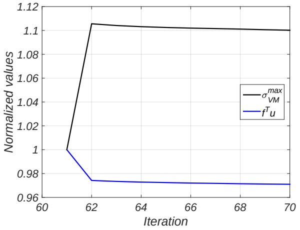
Adaptive $r_{min} = 11.28$	**Constant** $r_{min} = 9.00$
$\mathbf{f}^T\mathbf{u} = 2.434 \cdot 10^2$	$\mathbf{f}^T\mathbf{u} = 2.445 \cdot 10^2$
$\sigma_{VM}^{max} = 4.142 \cdot 10^{-1}$	$\sigma_{VM}^{max} = 4.393 \cdot 10^{-1}$

Adaptive filter radius





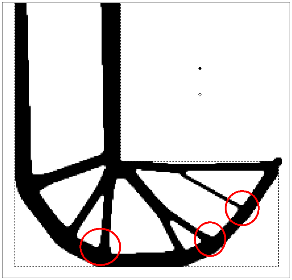




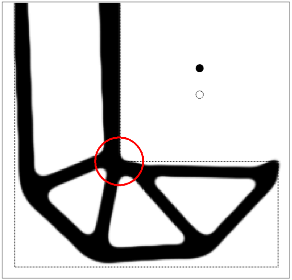
Looking at the required length scale

In some cases, compliance and stress require different length scales in different regions of the design:

Result with $r_{min} = 3$
low compliance



Result with $r_{min} = 9$
low max. stress



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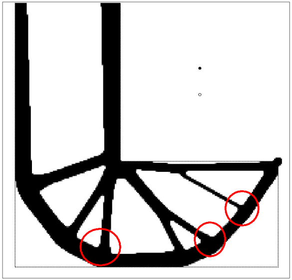

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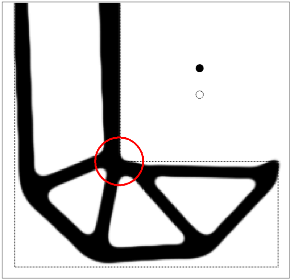

Question: how can separate length scales be accommodated such that compliance is minimized and stress constraints are satisfied?

A spatially varying filter radius

The length scale (controlled by filter radius) can be seen a spatially varying property:

- Define a critical “stress attractor” point
- Define an auxiliary function:

$$\phi(x,y) = \exp(-\left|\frac{d(x,y)}{D}\right|^{\theta}) \quad 0 \leq \phi(x,y) \leq 1$$

- Parameters: D is the characteristic influenced distance; θ determines the sharpness of $\phi(x,y)$

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- Parameters: D is the characteristic influenced distance; θ determines the sharpness of $\phi(x,y)$
- Spatial filter radius is defined as:

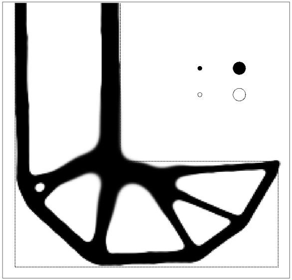
$$\hat{r}_{min}(x,y) = (1 + \gamma\phi(x,y))r_{min}$$

- Parameters: r_{min} is the native filter radius; γ is the increase in filter radius at the attractor point

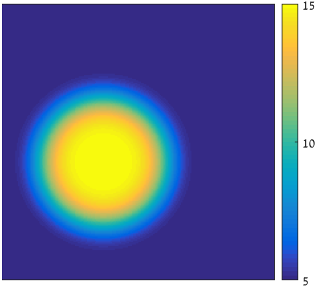
A spatially varying filter radius

Preliminary result with $r_{min} = 5$, $D = 50$, $\theta = 5$, $\gamma = 2$:

Layout

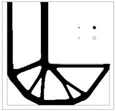
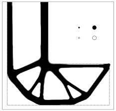
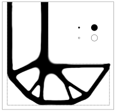
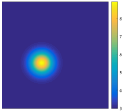
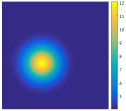
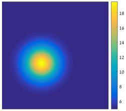


$\hat{r}_{min}(x, y)$



- Compliance $\mathbf{f}^T\mathbf{u} = 2.456 \cdot 10^2 \approx 2.445 \cdot 10^2 = \mathbf{f}^T\mathbf{u}(r_{min} = 9)$
- Max. stress $\sigma_{VM}^{max} = 3.153 \cdot 10^{-1} << 4.393 \cdot 10^{-1} = \sigma_{VM}^{max}(r_{min} = 9)$

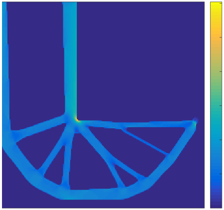
Spatially varying *and* adaptive filter radius

Initial / Final r_{min}	3.00 / 2.97	3.00 / 4.02	3.00 / 4.88
D / γ	20 / 2	30 / 2	30 / 3
$\sigma^{\star}_{max}$	$4.000 \cdot 10^{-1}$	$3.500 \cdot 10^{-1}$	$3.000 \cdot 10^{-1}$
σ^{max}_{VM}	$\mathbf{3.806 \cdot 10^{-1}}$	$\mathbf{3.346 \cdot 10^{-1}}$	$\mathbf{2.933 \cdot 10^{-1}}$
$\mathbf{f}^T\mathbf{u}$	$2.219 \cdot 10^{2}$	$2.262 \cdot 10^{2}$	$2.321 \cdot 10^{2}$
Layout			
$\hat{r}_{min}(x, y)$			

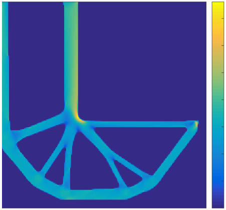
Spatially varying *and* adaptive filter radius

A look at the stress distributions:

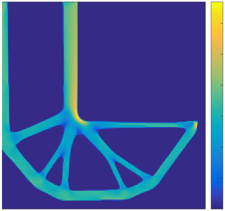
$r_{min} = 3.00$



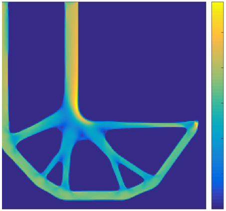
$r_{min} = 2.97,\ D = 20,\ \gamma = 2$



$r_{min} = 4.02,\ D = 30,\ \gamma = 2$



$r_{min} = 4.88,\ D = 30,\ \gamma = 3$



Spatially varying filter on-the-fly

In some cases, location of critical stress concentration is not known
$\Rightarrow$ **Identify and create** spatially varying filter

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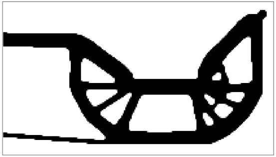
Minimum compliance optimization in two nested loops:

Set initial filter radius, then repeat:

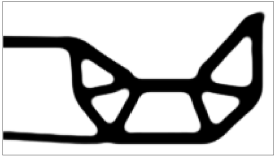
1. Standard minimum compliance (inner loop)
2. Find geometric locations of max. stress violations
3. Sort by stress magnitude
4. Remove duplicates / overlapping regions
5. Generate (a limited number of) auxiliary functions $\phi_i(x, y)$

Final example: U-bracket

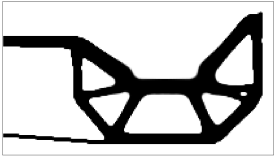
Constant $r_{min} = 3.00$
$\sigma_{VM}^{max} = 5.078 \cdot 10^{-1}$ $\mathbf{f}^T\mathbf{u} = 1.061 \cdot 10^2$



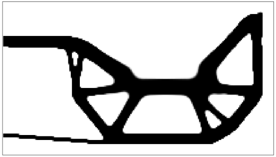
Adaptive $r_{min} = 3.00 \rightarrow 8.00$
$\sigma_{VM}^{max} = 4.125 \cdot 10^{-1}$ $\mathbf{f}^T\mathbf{u} = 1.291 \cdot 10^2$



Spatial $r_{min} = 3.00 \rightarrow 2.78$, $D = 20$, $\gamma = 2$
$\sigma_{VM}^{max} = 3.862 \cdot 10^{-1}$ $\mathbf{f}^T\mathbf{u} = 1.177 \cdot 10^2$



Spatial automatic $r_{min} = 3.00$, $D = 20$, $\gamma = 2$
$\sigma_{VM}^{max} = 3.509 \cdot 10^{-1}$ $\mathbf{f}^T\mathbf{u} = 1.174 \cdot 10^2$



Summary & conclusions

- Two approaches for satisfying stress constraints by **minimizing compliance with control on length scale:**
    - **Filter radius is a design variable**, determined according to stress constraint
    - **Filter radius varies spatially**, according to stress level
- For smooth stress distributions, **stresses are minimized together with compliance**
- Promising results – **reduction in maximum stresses**

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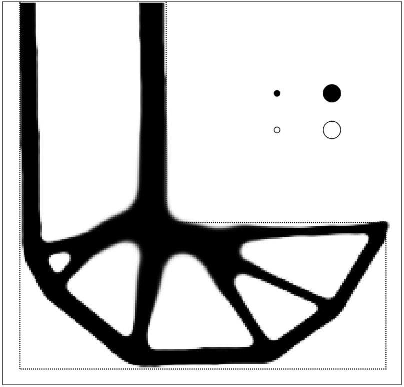
- Promising results – **reduction in maximum stresses**
- Future work: consistent sensitivity analysis, formally embed into optimization, robust control

QUESTIONS???

Extras

- It may not be necessary to symmetrize the filter operator:

$$\mathbf{f}^T\mathbf{u} = 2.460 \cdot 10^2 \qquad \sigma_{VM}^{max} = 3.133 \cdot 10^{-1}$$



Extras

- Area and curvature constraints using B-splines: