Minimum-cost topology and sizing optimization of viscous dampers for seismic retrofitting of 3-D frame structures

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Earthquakes

Devastating natural events which threaten lives, destroy property, and disrupt life-sustaining services and societal functions.

Gorkha earthquake, 2015 (Nepal). Killed $\cong 8,800$, injured $\cong 23,000$.

Conventional seismic design and retrofitting

New buildings

energy dissipation $\equiv$ plastic hinges

damage $\equiv$ costs

Existing buildings

costly & disruption of architectural features

Seismic protection systems

$\Rightarrow$ Fluid viscous dampers

$$F = c_d \dot{u}$$

Seismic retrofitting with fluid viscous dampers

Why optimization?

- Optimal dampers' distribution;
- Optimal dampers' size;
- Limit the variety of size-groups;
- Minimum cost & best performance.

Optimization approaches

Continuous approaches

[Gluck et al., 1996]
[Takewaki, 1997]
[Lavan and Levy, 2006]
and others...

- Optimal distributions and sizes of dampers;
- **Computational efficient**;
- **Effective** for **large scale** problems;
- Wide variety of damping coefficients.

Discrete approaches

[Zhang and Soong, 1992]
[Dargush and Sant, 2005]
[Kanno, 2013]
and others...

- **Practical distributions** and **sizes** of dampers;
- **Computationally robust**;
- Available dampers' sizes predefined;
- Computationally expensive for large scale problems.

Mixed-integer approaches

Damper **placement**, **sizing** and **selection**, discrete and continuous variables.

[Lavan and Amir, 2014]: minimum dampers' cost, inter-story drift constrained, dampers' distribution, selection, and sizing are variables of the problem, SLP with material interpolation functions;

[Pollini et al., 2014]: minimum realistic retrofitting cost, inter-story drift constrained, dampers' distribution, selection, and sizing are variables of the problem, GA;

Observations

- Most **realistic description** of the **problem**;
- **Practical** design **solutions**;
- **No aspect** of the design is **pre-defined**;
- Gradient-based: computationally efficient for large-scale problems, but not user-friendly;
- Zero order: computationally expensive for large-scale problems, but more user-friendly.

Goals of the current study

Approach for minimum-cost distributions of fluid viscous dampers combining concepts from **continuum topology** and **discrete material** optimization.

- Realistic retrofitting cost function;
- Formulation with discrete (topology and size selection) and continuous (sizes) variables;
- Final practical solutions with a reasonable computational effort.

Problem formulation

$$\min \text{COST} \quad \text{- retrofitting}$$

$$\text{s.t.} \quad d_i \leq \bar{d} \quad \text{- performance index}$$

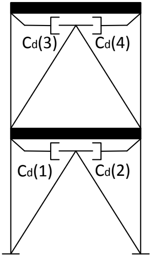
$$\text{with:} \quad \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_d(\tilde{\mathbf{c}}_d)]\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{e}a_g(t)$$

$$\mathbf{u}(0) = 0, \quad \dot{\mathbf{u}}(0) = 0$$

Variables:

$\mathbf{x}_1$, $\mathbf{x}_2$ discrete; y_1, y_2 continuous

- $\mathbf{x}_1(1) = 0$ no damper;
- $\mathbf{x}_1(1) = 1$, $\mathbf{x}_2(1) = 0$, $\mathbf{c}_d(1) = y_1\bar{c}_d$;
- $\mathbf{x}_1(1) = 1$, $\mathbf{x}_2(1) = 1$, $\mathbf{c}_d(1) = y_2\bar{c}_d$.

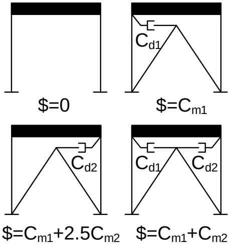


New realistic cost function

1. Cost due to the number of bays in which dampers are installed:

$$J_{bays} = \mathbf{x}_1^T \mathbf{C}_{mont}$$

$$\mathbf{C}_{mont} = D(\mathbf{C}_{m\mathbf{1}}) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} + D(\mathbf{C}_{m\mathbf{2}}) \begin{bmatrix} 0 \\ 1 + (1.5 + \frac{C_{m\mathbf{1}}}{C_{m\mathbf{2}}})(1 - \mathbf{x}_{\mathbf{1}}(1)) \\ 0 \\ 1 + (1.5 + \frac{C_{m\mathbf{1}}}{C_{m\mathbf{2}}})(1 - \mathbf{x}_{\mathbf{1}}(3)) \\ \vdots \end{bmatrix}$$



2. Cost of the dampers – $f(peak\ force \,\&\, stroke)$:

$$J_{dampers} = \bar{c}_d \mathbf{x}_1^T (y_1 \mathbf{1} + (y_2 - y_1)\mathbf{x}_2)$$

3. Cost due to the prototype testing – force-velocity behavior:

$$J_{sizes} = C_{type} \left[sgn(\mathbf{x}_1^T \mathbf{x}_2) + sgn(\mathbf{x}_1^T (\mathbf{1} - \mathbf{x}_2)) \right]$$

$$\Rightarrow J = J_{bays} + J_{dampers} + J_{sizes}$$

Mixed-integer formulation

$$\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{y}} \quad J \quad \textbf{(cost)}$$

$$\text{s. t.:} \quad d_{c,i} = \max_t(|d_i(t)/d_{all,i})|) \leq 1 \;\; \forall t, \forall i = 1, \ldots, N_{drifts} \quad \textbf{(drifts)}$$

$$x_{1,k} = \{0,1\} \;\; k = 1, \ldots, 2N_d \quad \textbf{(damper existence)}$$

$$x_{2,k} = \{0,1\} \;\; k = 1, \ldots, 2N_d \quad \textbf{(size-group association)}$$

$$0 \leq y_1^L \leq y_1 \leq y_1^U \leq y_2^L \quad (1^{st} \textbf{ size-group})$$

$$y_1^U \leq y_2^L \leq y_2 \leq y_2^U \leq 1 \quad (2^{nd} \textbf{ size-group})$$

$$\text{with:} \quad \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_d(\tilde{\mathbf{c}}_d)]\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{e}a_g(t) \; \forall t, \; \forall a_g(t) \in \mathcal{E}$$

$$\mathbf{u}(0) = 0, \; \dot{\mathbf{u}}(0) = 0$$

[Pollini et al., 2014]

Continuous formulation

$$\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{y}} \; J \quad \textbf{(cost)}$$

$$\text{s. t.:} \quad d_{c,i} = \max_t(|d_i(t)/d_{all,i})|) \le 1 \quad \forall t, \forall i = 1,\dots,N_{drifts} \quad \textbf{(drifts)}$$

$$0 \le x_{1,k} \le 1 \; k = 1,\dots,2N_d \quad \textbf{(existence)}$$

$$0 \le x_{2,k} \le 1 \; k = 1,\dots,2N_d \quad \textbf{(size-group association)}$$

$$0 \le y_1^L \le y_1 \le y_1^U \le y_2^L \quad (1^{st} \textbf{ size-group})$$

$$y_1^U \le y_2^L \le y_2 \le y_2^U \le 1 \quad (2^{nd} \textbf{ size-group})$$

$$\text{with:} \quad \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_d(\tilde{\mathbf{c}}_d)]\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{e}a_g(t) \, \forall t, \; \forall a_g(t) \in \mathcal{E}$$

$$\mathbf{u}(0) = 0, \; \dot{\mathbf{u}}(0) = 0$$

Ingredients

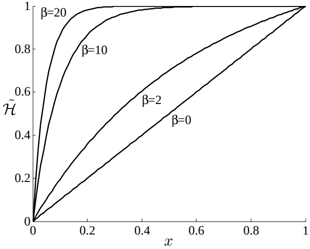
Material interpolation techniques (RAMP, [Stolpe and Svanberg, 2001]) - discrete solutions:

$$\tilde{c}_{d,j} = \bar{c}_d \boxed{\frac{x_{\mathbf{1},j}}{1+p(1-x_{\mathbf{1},j})}} \left(y_1 + (y_2 - y_1) \boxed{\frac{x_{\mathbf{2},j}}{1+p(1-x_{\mathbf{2},j})}} \right)$$

Continuously differentiable functions - gradient-based algorithm:

$$J_{sizes} = C_{type} \left[\boxed{sgn} (\mathbf{x}_1^T \mathbf{x}_2) + \boxed{sgn} (\mathbf{x}_1^T (\mathbf{1} - \mathbf{x}_2)) \right] \Rightarrow$$

$$d_{c,i} = \boxed{\max_t} \left(\left| d_i(t)/d_{all,i} \right) \right|$$



Constraint management - computational cost:

MMA: $\tilde{\mathbf{d}}_{c,active} = \left\{ \tilde{d}_{c,i} | \tilde{d}_{c,i} \geq 0.95 \right\}$

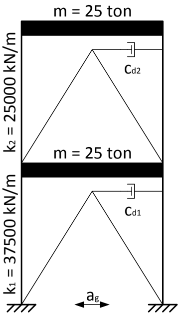
CPM: $\tilde{d}_{c,max} = \dfrac{\mathbf{1}^T D^{q+\mathbf{1}} \tilde{\mathbf{d}}_c(t_f) \mathbf{1}}{\mathbf{1}^T D^q \tilde{\mathbf{d}}_c(t_f) \mathbf{1}}$

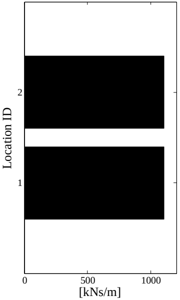
Notes regarding the algorithm

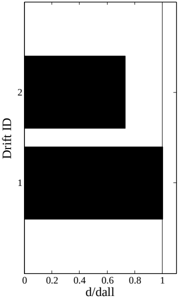
- Gradient-based algorithm:
  - Sequential convex programming (Method of Moving Asymptotes)
  - Sequential linear programming (Cutting Planes Method)
- Gradients of the constraints from an adjoint sensitivity analysis (additional time-history analyses with final conditions)
- Algorithm sensitive to:
  - Continuation scheme
  - Constraint aggregation/management
- Not yet user-friendly and computationally robust.

Simple case

1 damper size, $J = J_{dampers}$, LA02







DATA OF THE PROBLEM

$\bar{c}_d = 3000[kNs/m]$

$d_{all} = 0.009[m]$

RESULTS

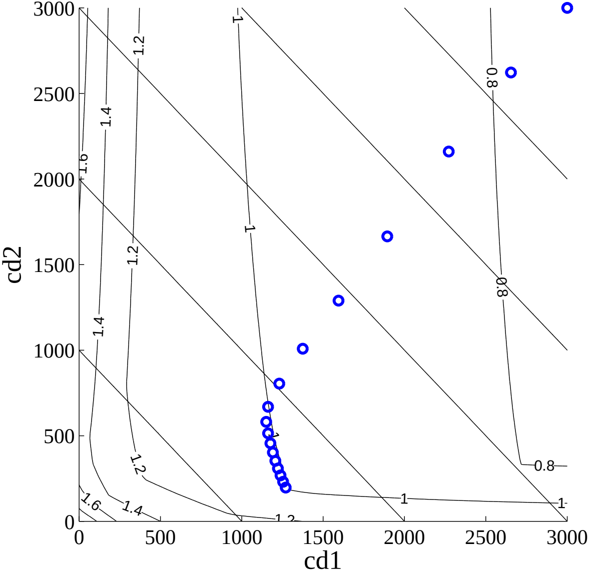
$c_{d1} = 1102.7\ [kNs/m]$

$c_{d2} = 1102.7\ [kNs/m]$

$J = 2203.69\ [kNs/m]$

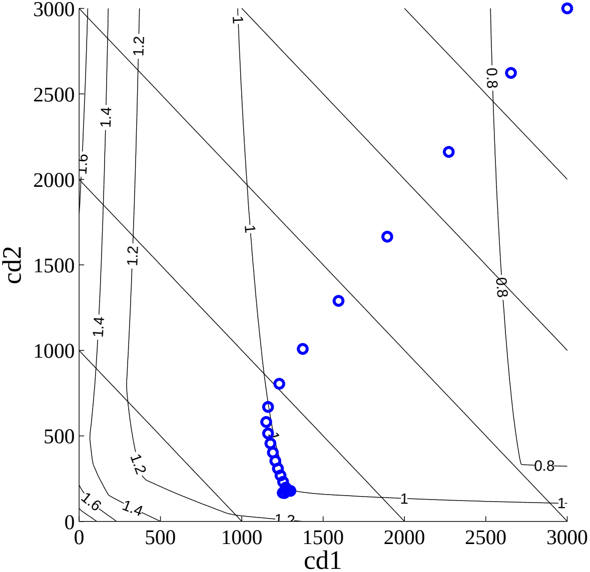
Simple case

Convergence to a discrete solution



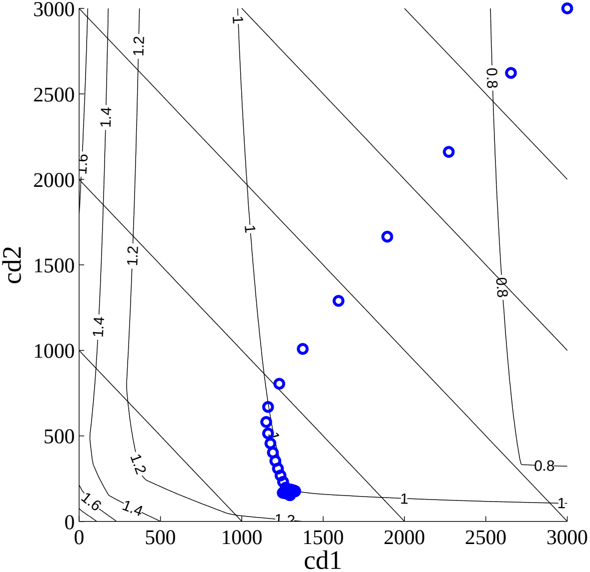
Simple case

Convergence to a discrete solution



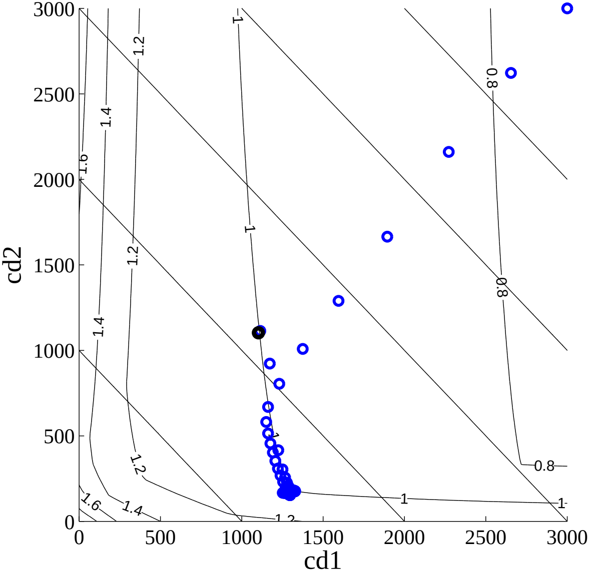
Simple case

Convergence to a discrete solution



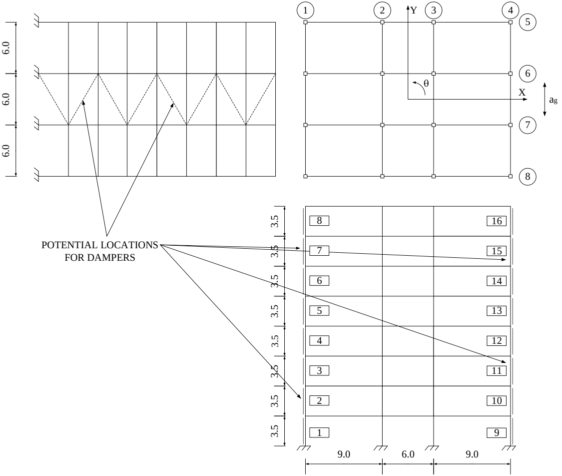
Simple case

Convergence to a discrete solution



Example 1

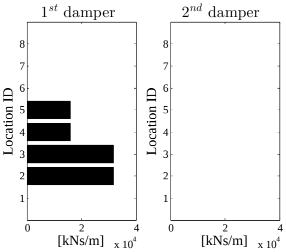
Eight-story asymmetric frame



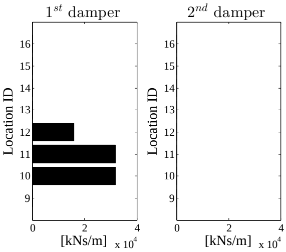
DATA OF THE STRUCTURE
columns: $0.5m \times 0.5m$ in frames 1 and 2
columns: $0.7m \times 0.7m$ in frames 3 and 4
beams: $0.4m \times 0.6m$
floor mass: $0.75[ton/m^2]$

DATA OF THE PROBLEM
$\bar{c}_d = 50000[kNs/m]$
$d_{all} = 0.035[m]$
$LA16$

Example 1 - results



MMA/GA. Locations 1 - 8.



MMA/GA. Locations 9 - 16.

	$\tilde{J},\ J$ [kNs/m]	$d_{c,max}/d_{all}$	$\bar{c}_{d1}$ [kNs/m]	$\bar{c}_{d2}$ [kNs/m]	Func. evaluations
MMA	334,656	1.0055 (0.55%)	15,933	31,735	$2923 \approx 10^{3.465}$
CPM	334,380	1.0072 (0.72%)	15,942	31,638	$2 \cdot 254 \approx 10^{2.706}$
GA	337,682	1 (0.00%)	15,906	32,490	$20 \cdot 1000 \cdot 110 \approx 10^{6.342}$

Conclusions

- **Material interpolation** techniques for optimal distribution and sizing of fluid viscous dampers;
- **Discrete** and practical solutions from a **continuum** formulation;
- Reasonable **computational cost**.

Thank you!

Further reading:

- Pollini N., Lavan O., and Amir O. (2014). *Towards realistic minimum-cost seismic retrofitting of 3D irregular frames using viscous dampers of a limited number of size groups*. 2ECEES, Istanbul, Turkey.
- Pollini N., Lavan O., and Amir O. (Submitted). *Towards realistic minimum-cost optimization of viscous dampers for seismic retrofitting*.