TECHNION
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Faculty of Civil and Environmental Engineering

Optimal design of skeletal structures with buckling considerations using nonlinear beam modeling

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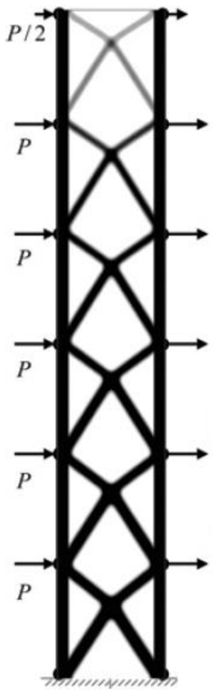
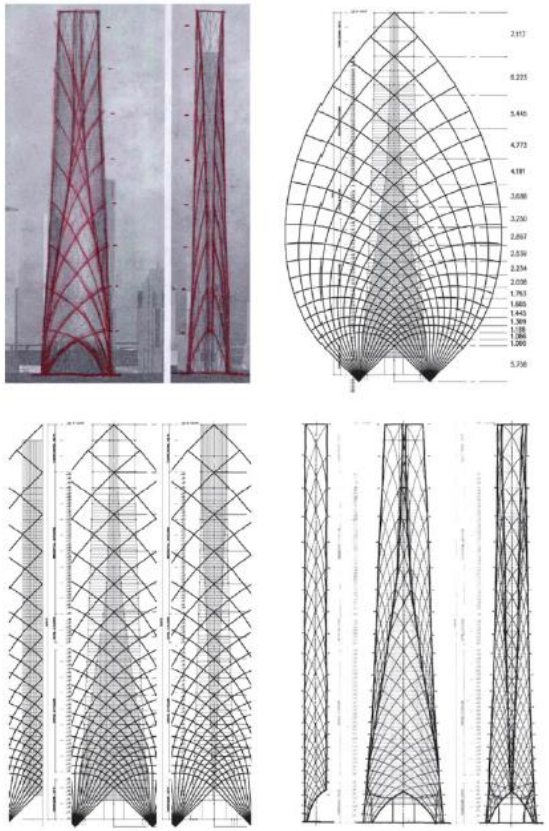
ECCOMAS Congress 2016, June 08 2016, Crete, Greece

Structural optimization in civil engineering

- Examples from Skidmore, Owings & Merrill (SOM)



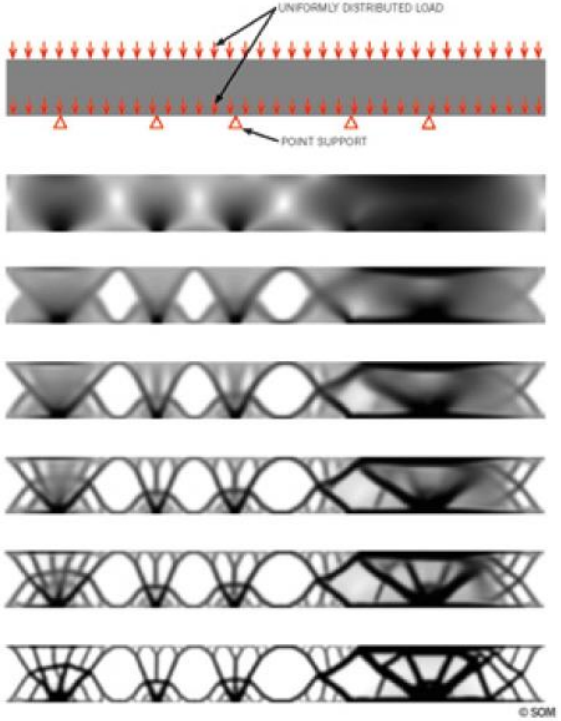
Beghini et al. 2013



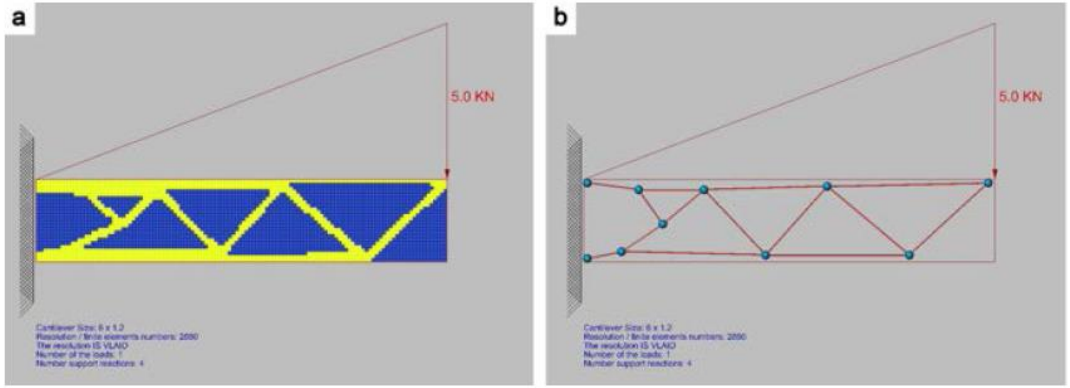
Besserud et al. 2013

Structural optimization in civil engineering

- Difficulties with interpreting complicated continuum forms
- Complicated conversion of continuum topology forms to practical construction



Besserud et al. 2013



Mostafavi et al. 2013

Utilizing innovative trends

- Robotic construction of optimized trusses
- Optimization based on:
  - Ground structure approach
  - Classical plastic design (LP) formulation

Fig. 57 robotic placing of bar member

“Practical” design

Design utilizing robotic capabilities

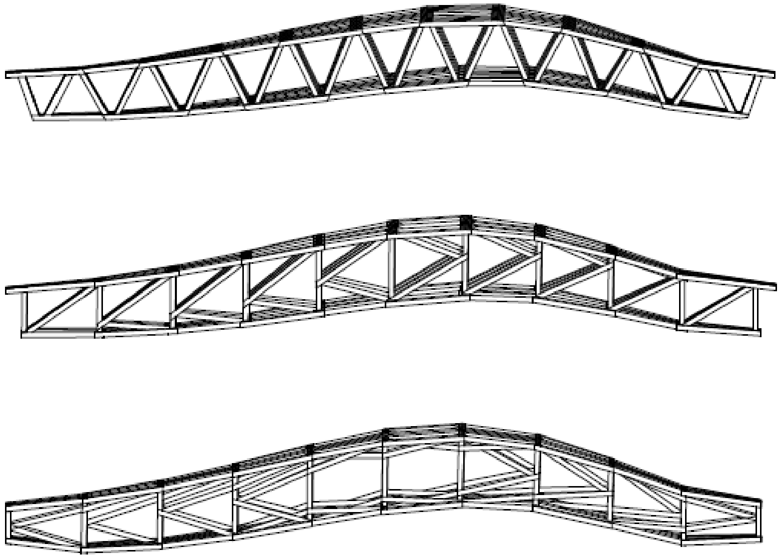


Fig. 55: 1:10 Scale model, K-truss

Truss optimization using ground structure approach

- Minimum weight / volume under stress constrains

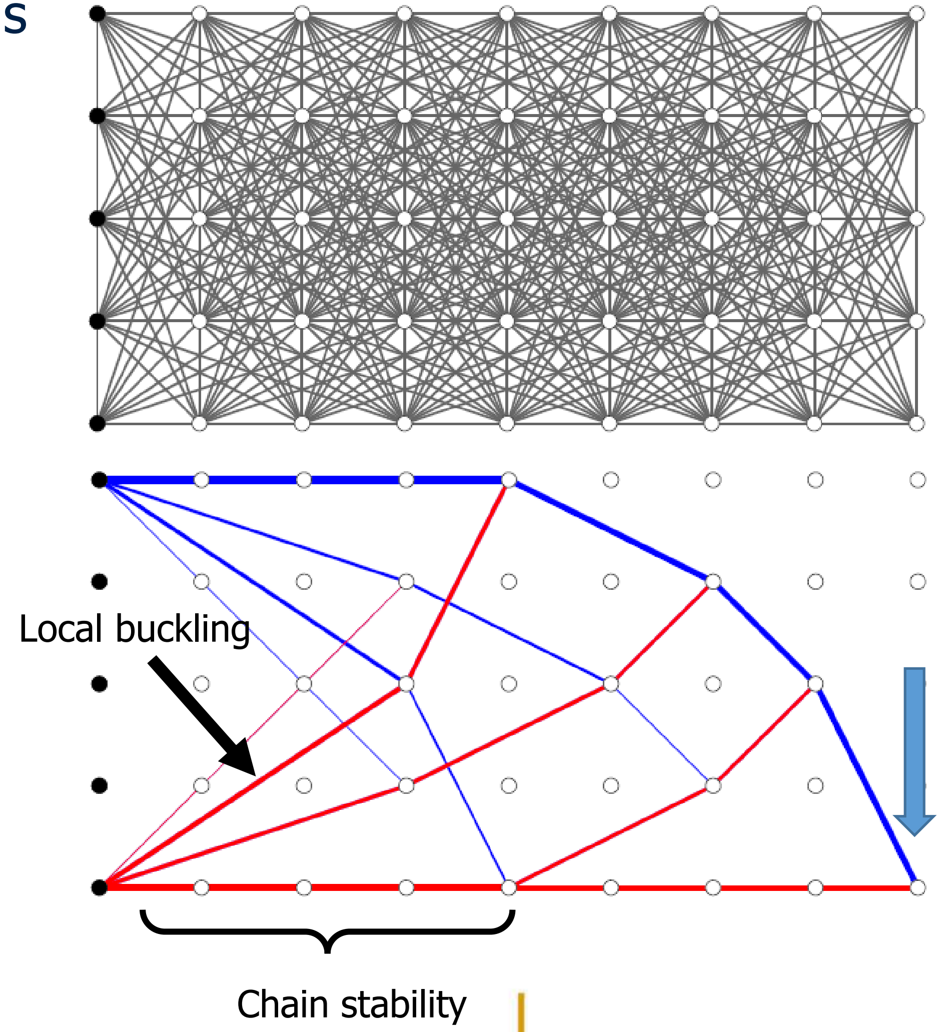
$$\min_{\mathbf{a},\mathbf{q}} \sum_{i=1}^{n} (a_i l_i)$$

$$s.t.: \quad \mathbf{Bq} = \mathbf{f}$$

$$a_i \sigma^{min} \leq q_i \leq a_i \sigma^{max}$$

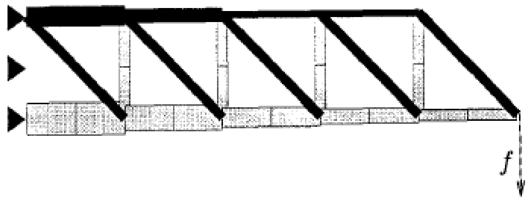
$$a_i \geq 0 \;, \quad i = 1..n$$

- Classical approach doesn’t take into account buckling considerations:
    - Local buckling
    - Global buckling
    - Chain stability



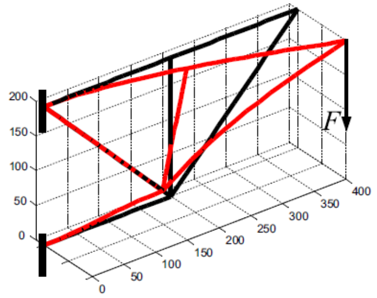
How are buckling considerations imposed in literature?

Achtziger 1999

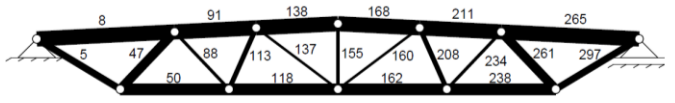


Ben-Tal et al. 2000,
Kocvara et al. 2002

Torii et. al. 2014



Mela 2013



(c) Minimum cost design, $C^* = 1202.29$ €. (Weight: $W^* = 1046.82$ kg)

- Plastic design problem - sequentially
  - Euler buckling (constraint on each bar)
  - Chain stability (constraint on each sequence of unbraced connected bars)
- Semi-definite problem
  - Global buckling (one stability constraint)
  - Chain stability (by overlapping bars)
- Eigen-value problem
  - Global buckling
  - Local stability (constraints)
- Mixed-integer LP
  - Local buckling (constraint)
  - Chain stability (overlapping bars)

The aim of the current work

- Account for all buckling considerations in a single formulation

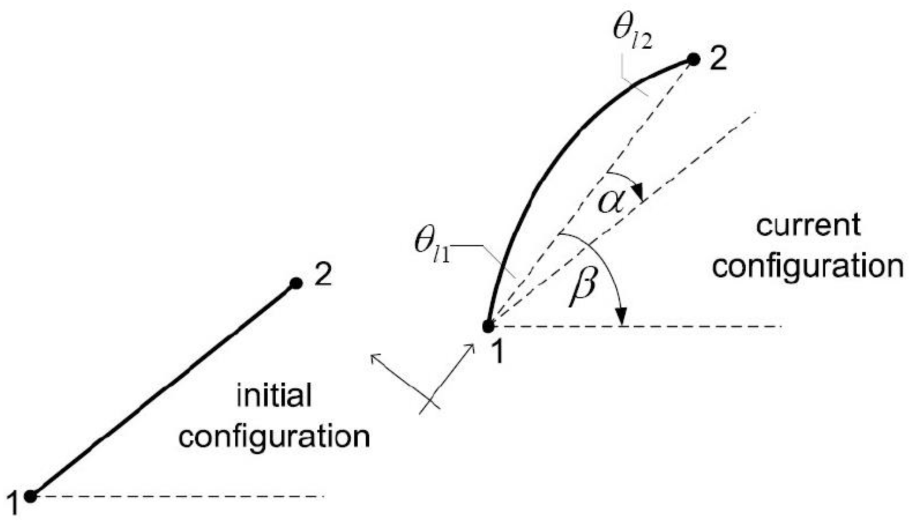
Main idea

- Use geometric nonlinear (GNL) beam formulation
- Optimize the response of the nonlinear structure, instead of imposing constraints

Geometric non-linear analysis

- GNL beam element derived using co-rotational formulation
- Kinematic assumptions:
  - Large displacements
  - Large rotations
  - Small strains

$$\mathbf{K}_t = \mathbf{B}^T \mathbf{C} \mathbf{B} + \frac{N}{l_n} \mathbf{z}\mathbf{z}^T + \frac{(M_1 + M_2)}{l_n^2} \left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T \right)$$



- Computational scheme: Newton-Raphson
  - Displacement control equilibrium

$$\mathbf{R}(\mathbf{u}, \theta) = \theta \hat{\mathbf{f}}_{\text{ext}} - \mathbf{f}_{\text{int}}(\mathbf{u}) = \mathbf{0}$$

Problem formulation - maxF

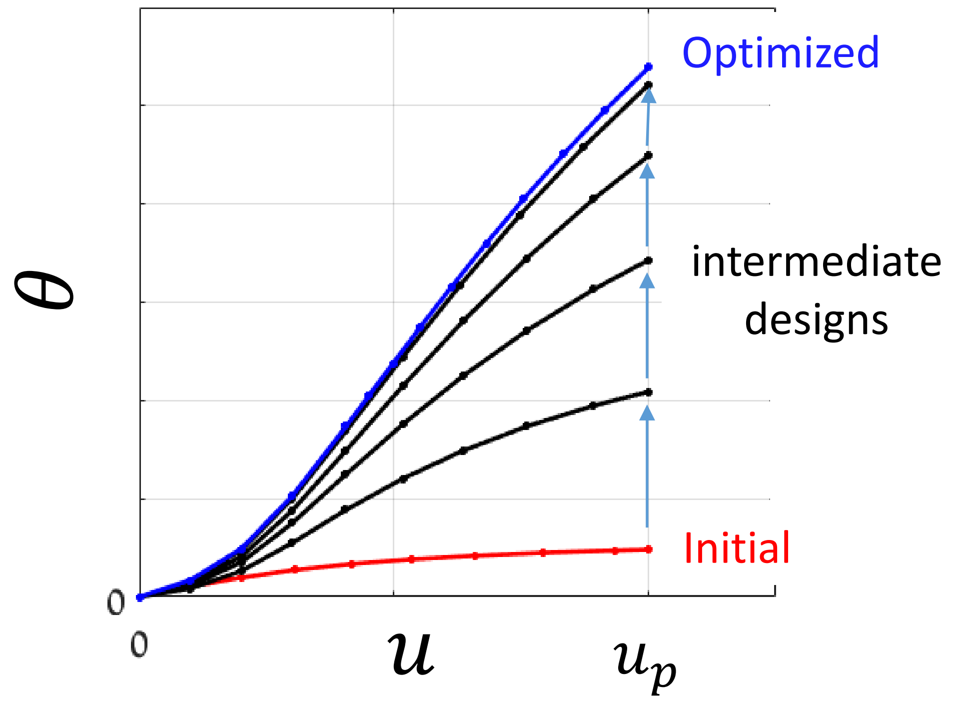
- Maximization of load-bearing capacity subject to a volume constraint with displacement control

$$\min_{\boldsymbol{\rho}} f = -\theta$$

$$s.t.: \quad g = \sum_{i=1}^{n} (\rho_i a l_i) \leq V^*$$

$$\rho_{min} \leq \rho_i \leq \rho_{max} \quad i = 1..n$$

$$\text{with: } \mathbf{R}(\boldsymbol{\rho}, \mathbf{u}, \boldsymbol{\lambda}) = \theta \hat{\mathbf{f}}_{\mathbf{ext}} - \mathbf{f}_{\mathbf{int}}(\boldsymbol{\rho}, \mathbf{u}) = \mathbf{0}$$



Problem formulation - minV

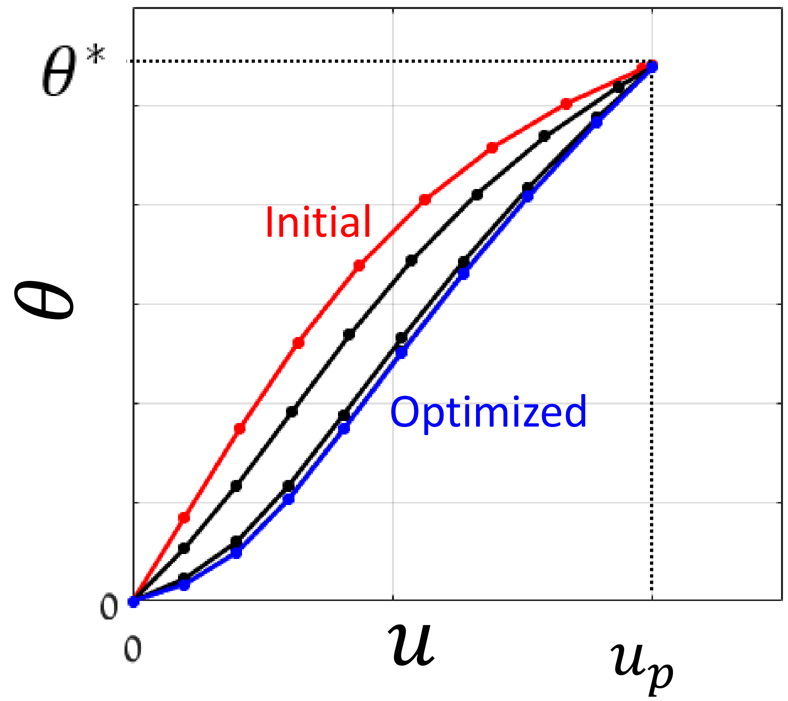
- Minimization of volume subject to a constraint on load-bearing capacity with displacement control

$$\min_{\boldsymbol{\rho}} f = V$$

$$s.t.: \quad g = \theta \geq \theta^*$$

$$\rho_{min} \leq \rho_i \leq \rho_{max} \quad i = 1..n$$

$$\text{with: } \mathbf{R}(\boldsymbol{\rho}, \mathbf{u}, \boldsymbol{\lambda}) = \theta \hat{\mathbf{f}}_{\mathbf{ext}} - \mathbf{f}_{\mathbf{int}}(\boldsymbol{\rho}, \mathbf{u}) = \mathbf{0}$$



Sensitivity analysis and solution

- Non-linear programming by the Method of Moving Asymptotes (MMA) – gradient based algorithm
- Sensitivity analysis following the adjoint method:

$$\hat{c}(\theta, \boldsymbol{\rho}, \mathbf{u}) = c(\theta, \boldsymbol{\rho}, \mathbf{u}) - \boldsymbol{\lambda}^T[\theta \mathbf{f}_{\mathbf{ext}} - \mathbf{f}_{\mathbf{int}}(\boldsymbol{\rho}, \mathbf{u})]$$

$$\frac{\partial \hat{c}}{\partial \rho_e} = \left(\frac{\partial c}{\partial \mathbf{u}} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}_{\mathbf{int}}}{\partial \mathbf{u}}\right)\frac{\partial \mathbf{u}}{\partial \rho_e} + \left(\frac{\partial c}{\partial \theta} - \boldsymbol{\lambda}^T \mathbf{f}_{\mathbf{ext}}\right)\frac{\partial \theta}{\partial \rho_e} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}_{\mathbf{int}}}{\partial \rho_e}$$

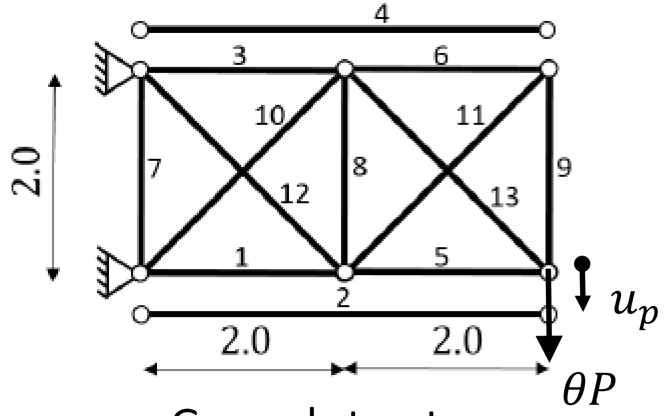
$$c(\theta, \boldsymbol{\rho}, \mathbf{u}) = -\theta$$

$$\frac{\partial \hat{c}}{\partial \rho_e} = \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}_{\mathbf{int}}}{\partial \rho_e}$$

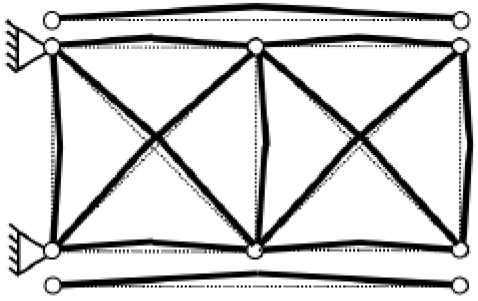
$$\begin{bmatrix} K_{ff} & K_{fp} \\ f_{ext}^f & f_{ext}^p \end{bmatrix} \begin{bmatrix} \lambda_f \\ \lambda_p \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial c}{\partial u_f} \\ \dfrac{\partial c}{\partial \theta} \end{bmatrix}$$

$$\begin{bmatrix} K_{ff} & K_{fp} \\ f_{ext}^f & f_{ext}^p \end{bmatrix} \begin{bmatrix} \lambda_f \\ \lambda_p \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

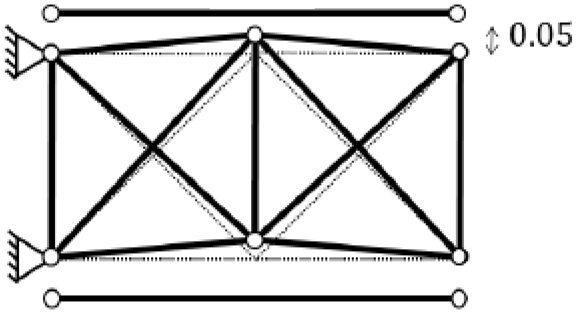
Demonstration: the effect of imperfections



Ground structure



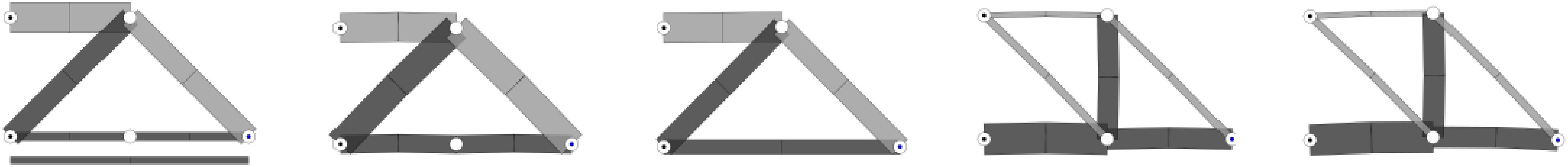
Local imperfection



Global imperfection

Prescribed disp.: Small u_p | Large u_p

Optimized layout:

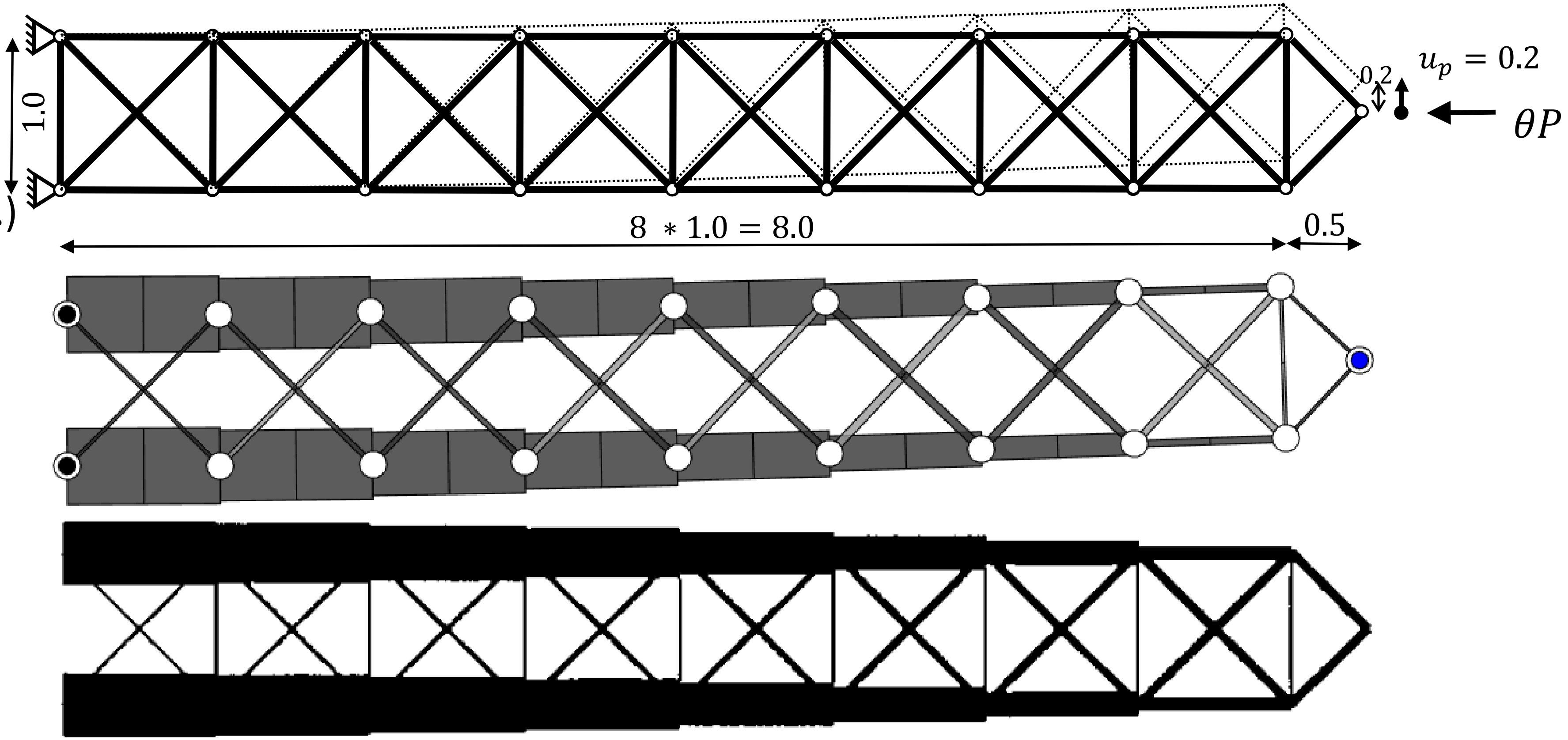


Imp.: [-] | Local only | Global only | Local only | Local and global

Preliminary results

- Global buckling of a cantilever – Ben-Tal et al. 2000

Ground structure (global imp.)



Proposed approach

Ben-Tal result

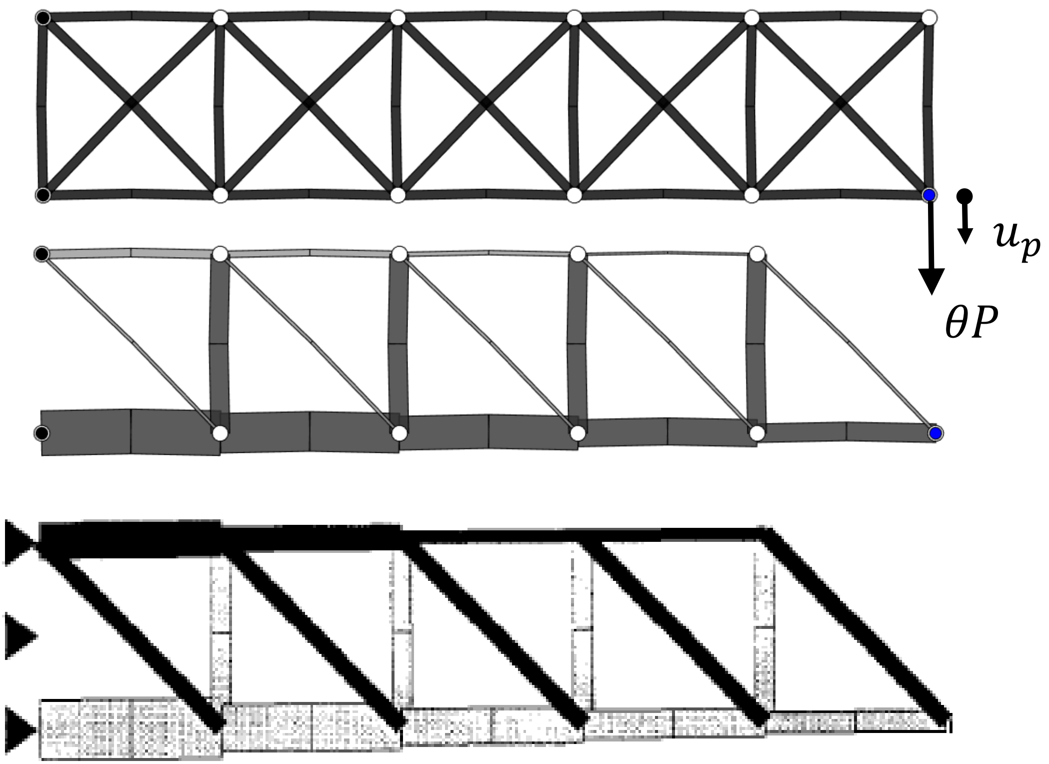
Preliminary results

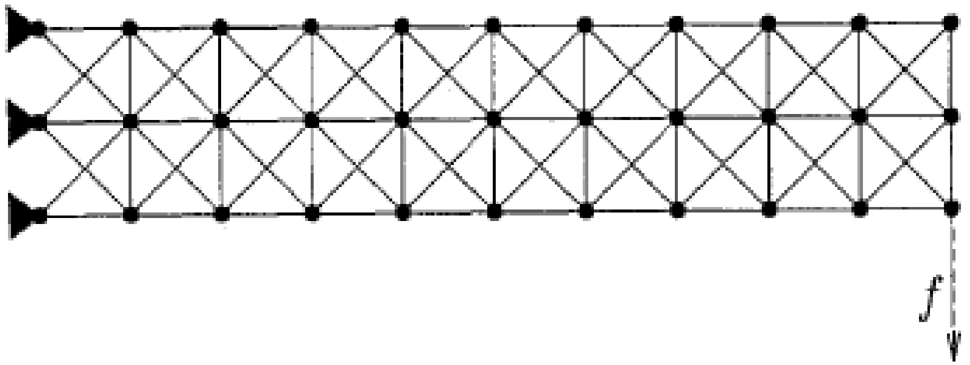
- Local stability of a cantilever – Achtziger 1999

Ground structure (local imp. + overlaps)

Proposed approach

Achtziger's result





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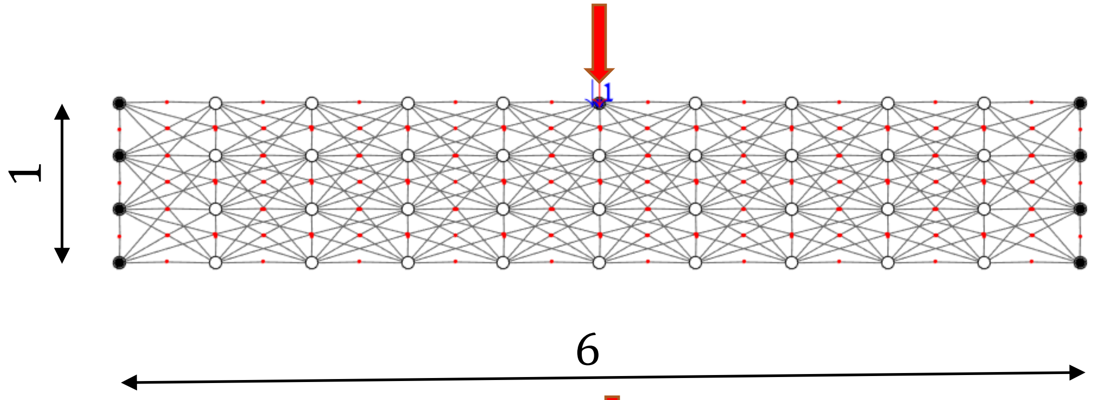
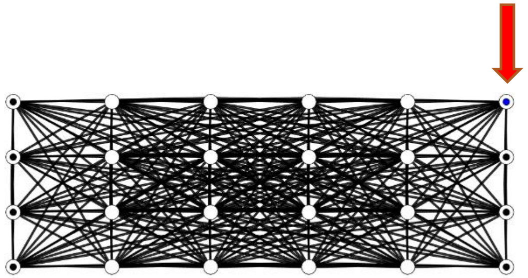
Ongoing work!

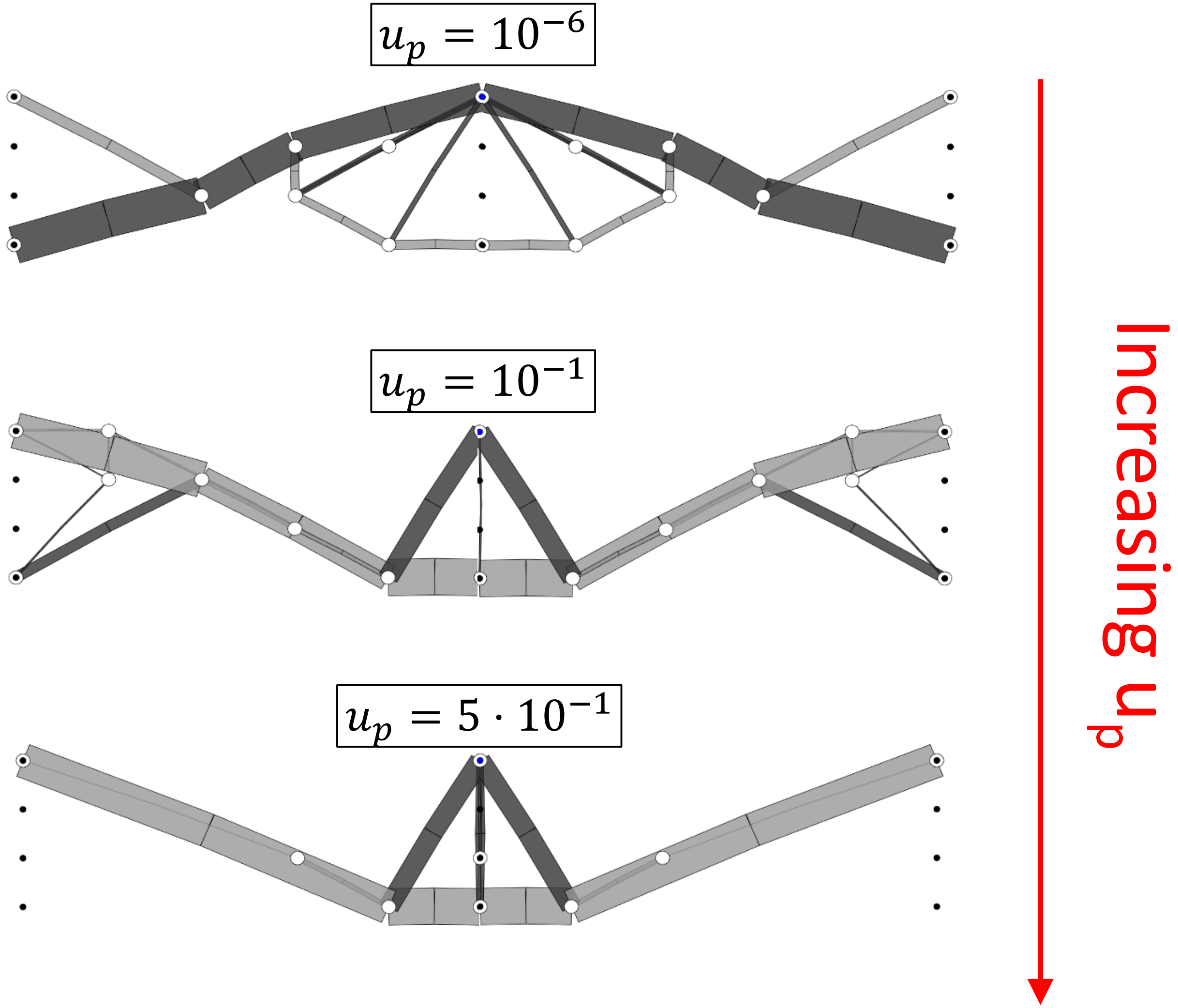
Preliminary results

- Double-clamped truss

Ground structure
with local imp. and overlaps







Summary

- GNL beam model has been integrated into truss optimization
- Considering buckling through the nonlinear response can replace the imposition of buckling constraints
- Preliminary results resemble those achieved by more traditional approaches involving constraints on local and global stability

Ongoing work

- Choice of local eccentricity – Euler buckling
- Choice of prescribed displacement
- Choice of global imperfection – initial positions of nodes
- Dealing with convergence difficulties for intermediate designs

Thank you!