Revisiting Approximate Reanalysis in Topology Optimization

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Outline

1. Background and motivation
2. Reanalysis in topology optimization
3. Revisiting by volume minimization
4. Extension to 3-D

Optimizing stiffness-to-volume trade-off

Focus is on classical problem statements, seeking the stiffest design:

- Minimize compliance s.t. constraint on volume / weight;
- Minimize volume / weight s.t. constraint on compliance.

Why are these important?

- Conceptual design phase of load-bearing components;
- Integrated analysis & design for architects and designers;
- Well-established - provide test cases for research and for formulating new procedures.

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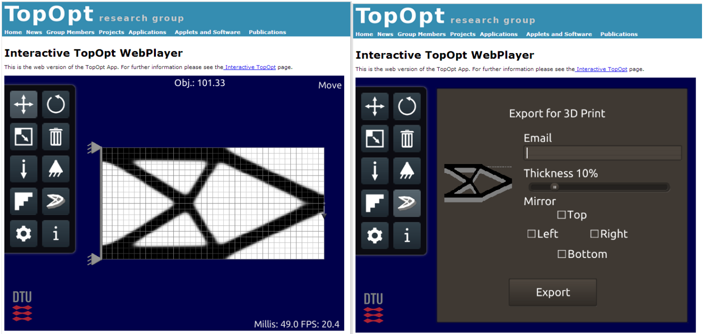
The need for efficient procedures

The aim: Improve computational efficiency.

The motivation:

- Increasing interest from architectural community $\rightarrow$ development of plug-ins and add-ons to CAD software;
- Interactivity is crucial! $\rightarrow$ a computational tool to "play" with.

Topopt App (Aage *et al.* 2013):



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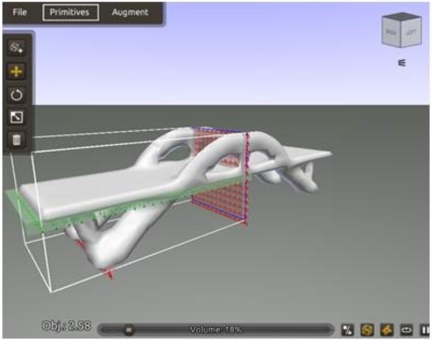

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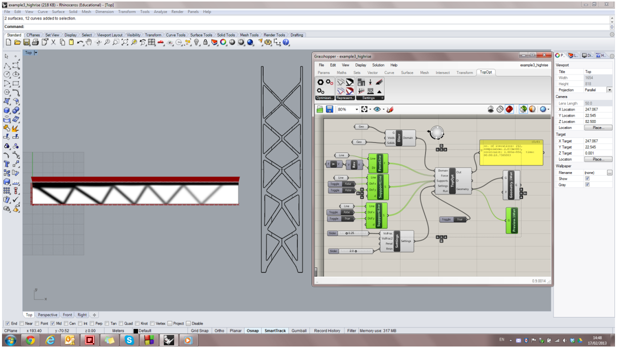
Rhino-Grasshopper component:



The need for efficient procedures

Approaches for reducing computational effort:

- **Multi-resolution / multi-scale:** e.g. Kim and Yoon 2000; Stainko 2006; MTOP - Nguyen *et al.* 2010, 2012; Guest and Smith Genut 2010;
- **Parallel procedures:** e.g. Borrvall and Petersson 2001; Kim *et al.* 2004; Vemaganti and Lawrence 2005; Mahdavi *et al.* 2006; Evgrafov *et al.* 2008; Aage and Lazarov 2013;
- **GPU implementation:** e.g. Wadbro and Berggren 2009; Schmidt and Schulz 2011; Suresh 2013; Zegard and Paulino 2013;
- **Recycling Krylov subspaces:** Wang *et al.* 2007.

Still room for advancements:

- High-resolution 2-D and 3-D are still very challenging on standard computers - main aim of current study;
- Insight also relevant for high-performance parallel environments.

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Problem formulation: minimum compliance

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\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & f_c = \mathbf{f}^T\mathbf{u} \\
\text{s.t.:} \quad & g_v = \sum_{e=1}^{N} v_e\bar{\rho}_e - V^\star \leq 0 \\
& 0 \leq \rho_e \leq 1 \qquad e = 1, \dots, N \\
\text{with:} \quad & \mathbf{K}(\bar{\boldsymbol{\rho}})\mathbf{u} = \mathbf{f}
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- $\bar{\boldsymbol{\rho}}$ represents filtered densities.
- Modified SIMP $E(\bar{\rho}) = E_{min} + (E_{max} - E_{min})\bar{\rho}^p$.
- Sensitivities are $\frac{\partial f_c}{\partial \bar{\rho}_e} = -\mathbf{u}^T \frac{\partial \mathbf{K}}{\partial \bar{\rho}_e}\mathbf{u}$.
- Solution obtained by optimality criteria or nonlinear programming.
- Computational cost dominated by solving $\mathbf{K}(\bar{\boldsymbol{\rho}})\mathbf{u} = \mathbf{f}$.

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\text{with:} \quad & \mathbf{K}(\bar{\rho})\widetilde{\mathbf{u}} \approx \mathbf{f}
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Following the Combined Approximations (CA) approach (Kirsch 1991):

Split the stiffness matrix:	$(\mathbf{K}_0 + \Delta\mathbf{K})\mathbf{u} = \mathbf{f}$
Introduce the recurrence:	$\mathbf{K}_0\mathbf{u}^k = \mathbf{f} - \Delta\mathbf{K}\mathbf{u}^{k-1}$
Expand the series:	$\mathbf{B} \equiv \mathbf{K}_0^{-1}\Delta\mathbf{K}$
	$\mathbf{u} = (\mathbf{I} - \mathbf{B} + \mathbf{B}^2 - \mathbf{B}^3 + \ldots)\mathbf{u}_0$
	$\mathbf{K}_0\mathbf{u}_0 = \mathbf{f}$
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Use a **few** terms as basis vectors:	$\widetilde{\mathbf{u}} = \mathbf{u}_1 y_1 + \mathbf{u}_2 y_2 + \ldots + \mathbf{u}_s y_s = \mathbf{R}_B\mathbf{y}$
Solve a reduced system:	$\mathbf{R}_B^T\mathbf{K}\mathbf{R}_B\mathbf{y} = \mathbf{R}_B^T\mathbf{f}$

Further reading: Monographs by Kirsch 2002, 2008; Amir *et al.* 2009

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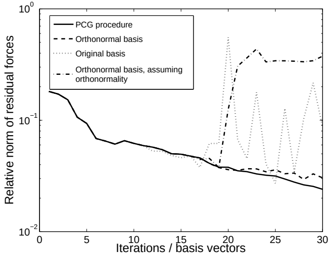
CA is a particular iterative solver

Kirsch, Kočvara and Zowe 2002: CA is a particular case of the PCG method - the preconditioner is the Cholesky factorization $\mathbf{K}_0 = \mathbf{U}_0^T \mathbf{U}_0$.

- "Reanalysis" ≈ "Recycled Preconditioning";
- Implementation as PCG is numerically stable:



Next step: blend reanalysis into robust formulations (multiple designs)...

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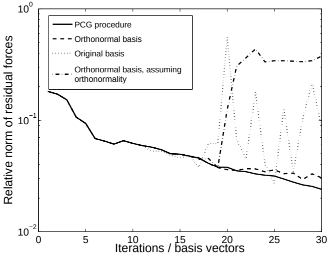
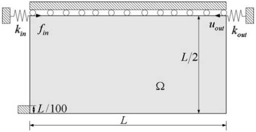


Next step: blend reanalysis into robust formulations (multiple designs)...

Robust formulations in topology optimization

- Manufacturing errors pose a challenge in the design of micro mechanisms:



- More robust design can be achieved by e.g. a worst-case approach with multiple designs accounting for uniform errors:

(a) Eroded design
$f = -2.15,\ f_v = 0.250,\ M_{nd} = 0.18$

(b) Intermediate design
$f = -2.30,\ f_v = 0.300,\ M_{nd} = 0.49$

(c) Dilated design
$f = -2.15,\ f_v = 0.347,\ M_{nd} = 0.23$

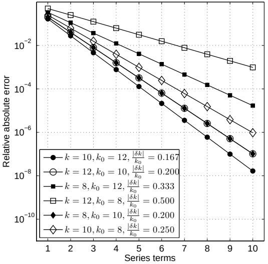
Further reading: Sigmund 2009; Wang *et al.* 2011; Schevenels *et al.* 2011

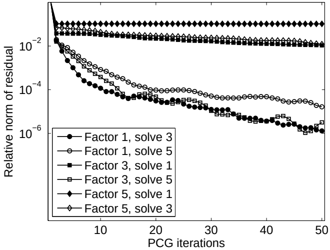
Reanalysis is better with a stiffer preconditioner

Examining Kirsch's CA series:

$$\mathbf{u} = \left(\mathbf{I} - \left(\mathbf{K}_0^{-1}\Delta\mathbf{K}\right) + \left(\mathbf{K}_0^{-1}\Delta\mathbf{K}\right)^2 - \left(\mathbf{K}_0^{-1}\Delta\mathbf{K}\right)^3 + \ldots\right)\mathbf{u}_0$$

- If $\mathbf{K}_0 \succ \mathbf{K}$ then convergence is guaranteed;
- For any pair of designs, convergence is faster if the stiffer one plays the role of the preconditioner.





Further reading: Amir *et al.* 2012

(1=dilated; 3=intermediate; 5=eroded)

Revisiting reanalysis in stiffness-to-volume procedures

During optimization iterations:

- Minimum compliance s.t. volume constraint: Design is **stiffened** while approaching the allowable amount of material;
- Minimum volume s.t. compliance constraint: Material is removed while approaching the compliance requirement $\rightarrow$ design is typically **softened**.

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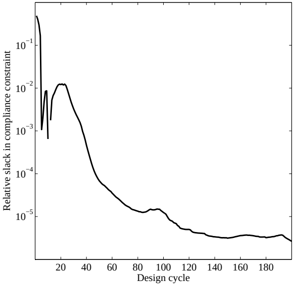

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$$\widetilde{\mathbf{u}}_k \approx \left(\mathbf{I} - \mathbf{K}\left(\bar{\rho}_{k-l}\right)^{-1}\Delta\mathbf{K} + \left(-\mathbf{K}\left(\bar{\rho}_{k-l}\right)^{-1}\Delta\mathbf{K}\right)^2 + \left(-\mathbf{K}\left(\bar{\rho}_{k-l}\right)^{-1}\Delta\mathbf{K}\right)^3 + \ldots\right)\mathbf{u}_{k-l}$$

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Necessary building block: OC procedure

Minor obstacle:

- Min. compliance s.t. volume: OC uses bi-section scheme, constraint is linear - no need to re-evaluate for every inner design update;
- Min. volume s.t. compliance: Constraint is nonlinear, needs to be evaluated for every inner design update $\rightarrow$ not very efficient...

Linear approximation of the compliance constraint:

$$\widetilde{g}_c(\Lambda) = g_c^k + \sum_{e=1}^{N} \left.\frac{\partial g_c}{\partial \rho_e}\right|_{\rho_e^k} \left(\rho_e^{k+1}(\Lambda) - \rho_e^k\right) \approx 0$$

Reciprocal approximation of the compliance constraint:

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Results: 2-D toy problem

Min. volume leads to **fewer PCG iterations** and **smaller errors**:

Formulation	Objective	Normalized constraint	Matrix factorizations	PCG iterations
Min. compliance, standard	$f_c = 136.1$	$g_v = 3.16 \cdot 10^{-8}$ $V^\star = 0.35 \times N$	200	—
Min. volume, standard	$f_v = 0.35$	$g_c = -2.65 \cdot 10^{-6}$ $c^\star = 136.1$	200	—
Min. compliance, reanalysis	$f_c = 136.0$	$g_v = 7.21 \cdot 10^{-7}$ $V^\star = 0.35 \times N$	25	**565**
Min. volume, reanalysis	$f_v = 0.35$	$g_c = -3.11 \cdot 10^{-6}$ $c^\star = 136.1$	22	**369**

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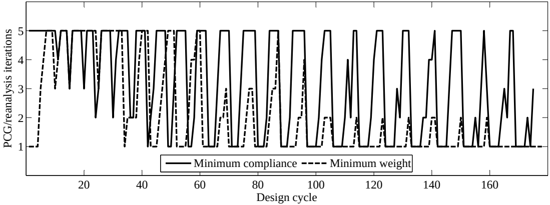

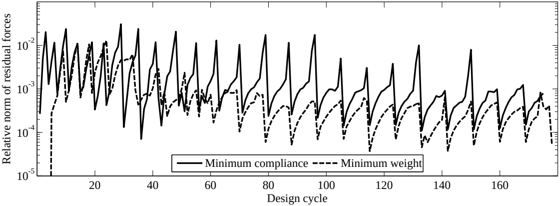

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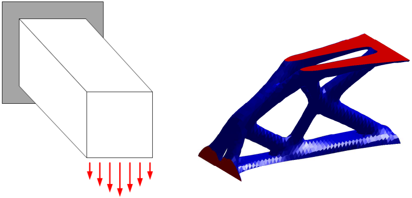
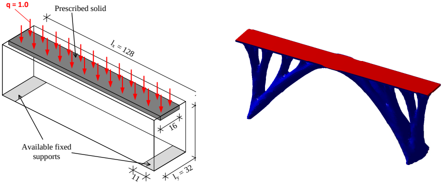
Extension to 3-D using MGCG

- Matrix factorization is impractical for 3-D problems;
- Multigrid-PCG (MGCG) is used as the accurate solver;
- MGCG exhibits mesh-independent convergence, even for high contrast topologies;
- "Reanalysis" is replaced by recycling the multigrid preconditioner.



$80 \times 40 \times 40$ elements, $4.1 \cdot 10^5$ DOF



$64 \times 16 \times 48$ elements, $1.6 \cdot 10^5$ DOF

Further reading: Tatebe and Oyanagi 1994, Ashby and Falgout 1996, Amir *et al.* 2014

Relaxed MGCG convergence

- Alternative stopping criteria based on value of target functional:

$$\frac{\left|\mathbf{f}^T\widetilde{\mathbf{u}}_i - \widetilde{\mathbf{u}}_i^T\mathbf{K}\widetilde{\mathbf{u}}_i\right|}{\widetilde{\mathbf{u}}_i^T\mathbf{K}\widetilde{\mathbf{u}}_i} < \epsilon$$

- Arioli's stopping criteria related to FE discretization error:

$$\xi_i \leq \eta^2\left(\widetilde{\mathbf{u}}_i^T\mathbf{r}_0 + \mathbf{f}^T\widetilde{\mathbf{u}}_0\right)$$

- Direct monitoring of the design sensitivities:

$$\frac{\left|\widetilde{\mathbf{u}}_i^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_i - \widetilde{\mathbf{u}}_{i-1}^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_{i-1}\right|}{\left|\widetilde{\mathbf{u}}_i^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_i\right|} < \epsilon \qquad \forall e$$

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$$\xi_i \leq \eta^2\left(\widetilde{\mathbf{u}}_i^T\mathbf{r}_0 + \mathbf{f}^T\widetilde{\mathbf{u}}_0\right)$$

- Direct monitoring of the design sensitivities:

$$\frac{\left|\widetilde{\mathbf{u}}_i^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_i - \widetilde{\mathbf{u}}_{i-1}^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_{i-1}\right|}{\left|\widetilde{\mathbf{u}}_i^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_i\right|} < \epsilon \qquad \forall e$$

$$\left|\widetilde{\mathbf{u}}_i^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_i - \widetilde{\mathbf{u}}_{i-1}^T\frac{\partial\mathbf{K}}{\partial\bar{\rho}_e}\widetilde{\mathbf{u}}_{i-1}\right| < \left|\frac{\partial f_v}{\partial\rho_e} + \Lambda\frac{\partial g_c}{\partial\rho_e}\right| \qquad \forall\left\{e|0 < \rho_e < 1\right\}$$

Further reading: Amir *et al.* 2010, Arioli 2004, Amir *et al.* 2014

Efficiency in 3-D

$80 \times 40 \times 40$ cantilever, 50 design cycles

Procedure	Objective	Constraint	MGCG/PCG it.	MATLAB time
MinC-ACC	$f_c = 5,562$	$V^\star = 0.120 \times N$	2,535	**1,053.31**
MinV-ACC	$f_v = 0.114$	$c^\star = 5,562$	2,005	896.05
MinV-RE5	$f_v = 0.114$	$c^\star = 5,562$	956	**685.08**
MinV-MFRE5	$f_v = 0.114$	$c^\star = 5,562$	900	**544.91**
MinV-SM-MFRE5	$f_v = 0.114$	$c^\star = 5,562$	440	**484.71**
MinC-ICPCG	$f_c = 5,560$	$V^\star = 0.120 \times N$	34,174	**4,974.98**

$64 \times 16 \times 48$ bridge, 50 design cycles

Procedure	Objective	Constraint	MGCG/PCG it.	MATLAB time
MinC-ACC	$f_c = 4.326 \cdot 10^5$	$V^\star = 0.1 \times N$	2,168	**354.78**
MinV-ACC	$f_v = 0.0969$	$c^\star = 4.326 \cdot 10^5$	1,791	320.13
MinV-MFRE5	$f_v = 0.0967$	$c^\star = 4.326 \cdot 10^5$	942	**210.49**
MinV-SM-MFRE5	$f_v = 0.0968$	$c^\star = 4.326 \cdot 10^5$	346	**176.12**

Summary

- An efficient procedure for continuum structural topology optimization was presented;
- Computational time is reduced by exploiting "stiff" preconditioning in reanalysis-based optimization;
- Reanalysis concepts applicable to 2-D problems are extended to 3-D in the form of recycled preconditioning within a general MGCG framework;
- Run time of the minimum volume procedure was roughly twice faster than that of a standard minimum compliance procedure;
- No compromise on the quality of the results in terms of the compliance-to-weight trade-off;
- A step towards the effective integration of 3-D topology optimization into CAD software and mobile applications.

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- Full paper available online in SMO;
- MATLAB codes for 2-D are freely available on my webpage, 3-D upon request.

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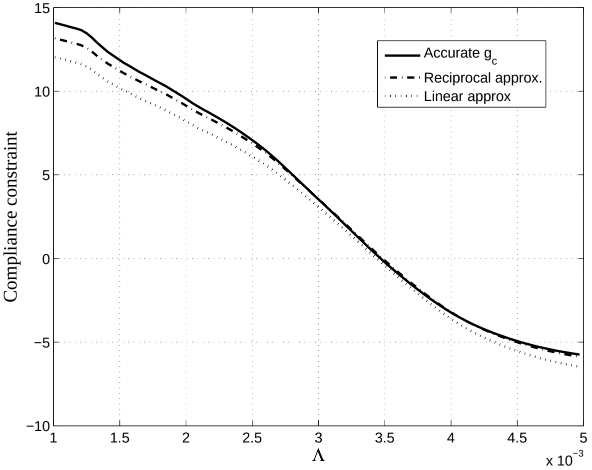
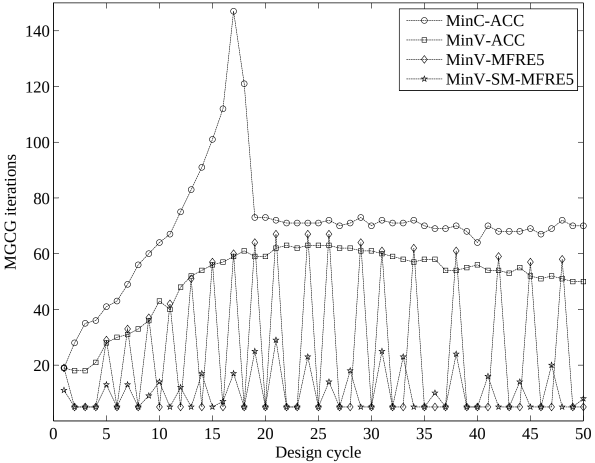
QUESTIONS?

Approximating $g_c(\Lambda)$



MGCG iterations



MGCG iterations